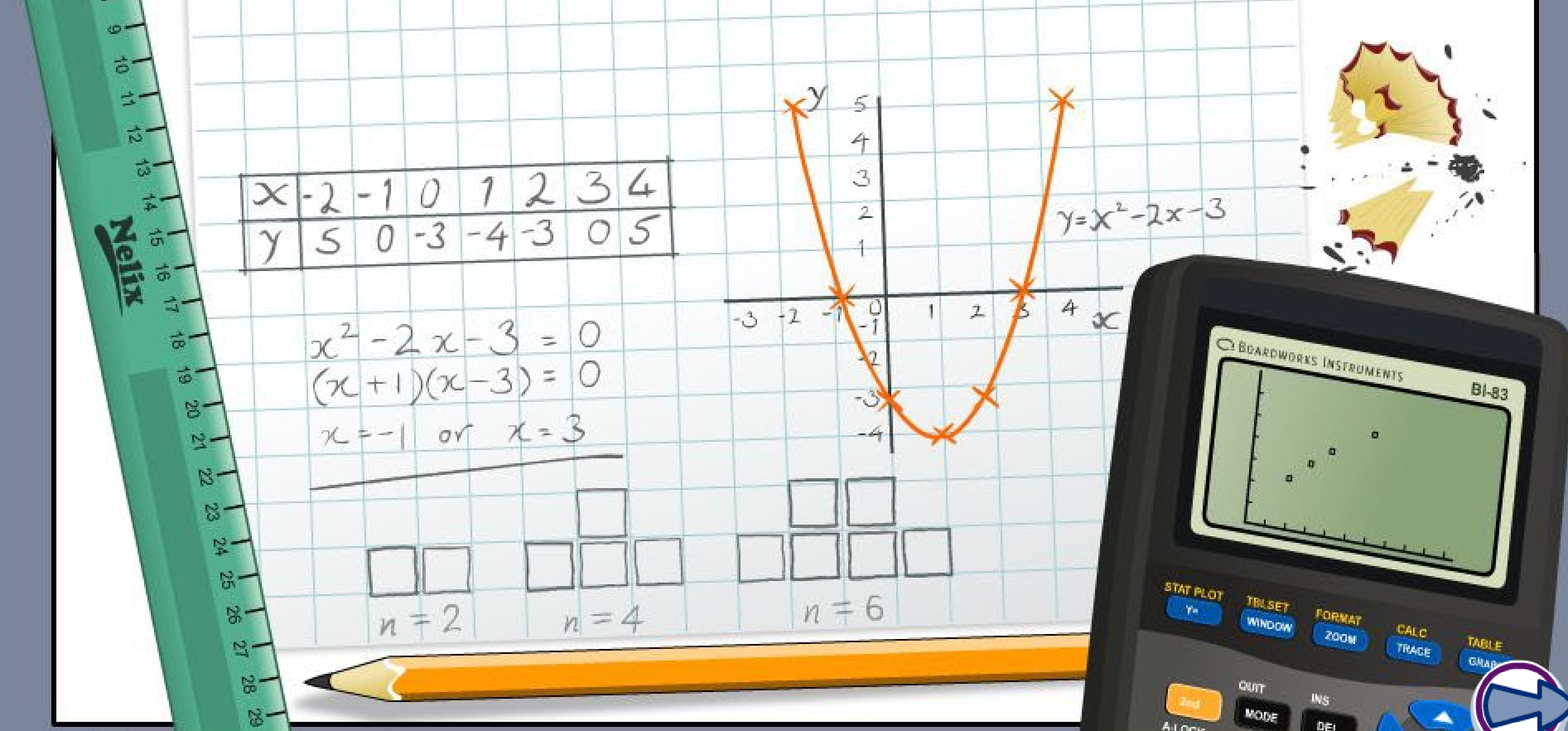
Exponent laws



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

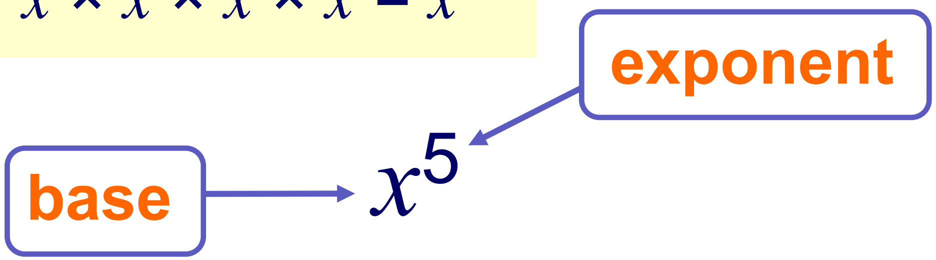
This icon indicates teacher's notes in the Notes field.

Exponent notation

How can we simplify $x \times x \times x \times x \times x$?

We use **exponent notation** as shorthand for multiplication by the same number.

For example: $x \times x \times x \times x \times x = x^5$



This number is read as “x to the fifth power”.

$y \times y \times y$	$= y^3$	“y to the third power” or “y cubed”
$z \times z$	$= z^2$	“z to the second power” or “z squared”
$q \times q \times q \times q$	$= q^4$	“q to the fourth power”

Simplifying using exponent notation

Using your knowledge of exponent notation, how would you simplify the following expressions?

$3p \times 2p = 3 \times p \times 2 \times p = \mathbf{6p^2}$

$q^2 \times q^3 = q \times q \times q \times q \times q = \mathbf{q^5}$

$3r \times r^2 = 3 \times r \times r \times r = \mathbf{3r^3}$

$3t \times 3t = (3t)^2$

$= 3 \times 3 \times t \times t$

$= \mathbf{9t^2}$

Work through the following multiplications:

$a^4 \times a^2$ $b^3 \times b^7$ $c \times c^5$

Can you identify a rule for when we multiply two terms containing exponents that have the same base?

$$a^4 \times a^2 = (a \times a \times a \times a) \times (a \times a)$$

$$= a \times a \times a \times a \times a \times a$$

$$= \mathbf{a^6}$$

4 + 2 = 6

When multiplying two terms with the base, the exponents can be added.

In general, $x^m \times x^n = x^{(m+n)}$

Drag each expression to the appropriate box.

x^3y^4	x^5y^4

$yxyxyxy$

Marking homework

Sid's homework is shown below. His teacher has marked one question. Can you mark the rest? Explain any of his mistakes and give the correct answers.

1) $a^4 \text{ x } a^2 = a^8$ ✗ $a^4 \times a^2 = a^{4+2} = \underline{a^6}$

2) $b^3 \text{ x } b^2 = b^5$ ✓

3) $3a^2 \text{ x } 3a^5 = 6a^7$ ✗ $= (3 \times 3) \times (a^2 \times a^5) = 9a^{2+5}$
$= \underline{9a^7}$

4) $4b^2 \text{ x } 2b^3 = 8b^6$ ✗ $= (4 \times 2) \times (b^2 \times b^3) = 8b^{2+3}$
$= \underline{8b^5}$

5) $5y^2 \text{ x } 4y^4 = 20y^6$ ✓

Dividing numbers in exponent form

When we divide two numbers written in exponent form that have the same base, we can see another interesting result.

For example:

$$4^5 \div 4^2 = \frac{\overset{1}{\cancel{4}} \times \overset{1}{\cancel{4}} \times 4 \times 4 \times 4}{\overset{1}{\cancel{4}} \times \overset{1}{\cancel{4}}} = 4 \times 4 \times 4 = 4^3 = 4^{(5-2)}$$

$$5^6 \div 5^4 = \frac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}} \times 5 \times 5}{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}}} = 5 \times 5 = 5^2 = 5^{(6-4)}$$

When dividing two terms with the same base, the exponents can be subtracted.

In general,

$$x^m \div x^n = x^{(m-n)}$$

Dividing exponents

Drag the simplified expressions to their equivalents.

$\frac{a \times a \times a \times a \times a}{a} =$

$a^6 \div a^4 =$

$4a^7 \div 2a^5 =$

$15a^4 \div 3a^2 =$

$10a^{10} \div 2a^5 =$

$8a^{12} \div 4a^8 =$

a^4

S C ?

Expressions of the form $(x^m)^n$

Sometimes we raise a power to another power. For these terms, in the form $(x^m)^n$, the m and n can be multiplied.

For example,

$(y^3)^2 = y^3 \times y^3$

$= (y \times y \times y) \times (y \times y \times y)$

$= \boldsymbol{y^6}$

$(q^2)^4 = q^2 \times q^2 \times q^2 \times q^2$

$= q^{(2 + 2 + 2 + 2)}$

$= \boldsymbol{q^8}$

In general, the rule when dealing with a single term raised to a power, then raised to another power, is:

$$\boldsymbol{(x^m)^n = x^{mn}}$$

Expressions of the form $(xy)^n$

When a term is in the form $(xy)^n$ the n can be applied to both the x and y.

For example,

$$(xy)^3 = xy \times xy \times xy$$
$$= (x \times x \times x) \times (y \times y \times y)$$
$$= \mathbf{x^3y^3}$$

$$(pq^2)^4 = pq^2 \times pq^2 \times pq^2 \times pq^2$$
$$= p^{(1+1+1+1)}q^{(2+2+2+2)}$$
$$= \mathbf{p^4q^8}$$

In general, the rule when dealing with two or more terms raised to a power, then raised to another power, is:

$$\mathbf{(xy)^n = x^ny^n}$$

Parentheses and exponents

Drag the expressions to the appropriate box.

a^6b^4	a^4b^9

$a^4(b^3)^3$

The power of 1

Find the value of the following using your calculator:

6^1 47^1 0.9^1 $–5^1$ 0^1

Any number raised to the power of 1 is equal to the number itself.

e.g. $2^1 = 2$

$a^1 = a$,

$–3.5^1 = –3.5$

In general, $$x^1 = x$$

Because of this we don't usually write the power when a number is raised to the power of 1.

The zero exponent

It is possible to write a term to the power of zero.

Use the division rule $x^m \div x^n = x^{(m-n)}$ to write an equivalent expression for x^0. Can you use this result to figure out the value of x^0?

Using the division rule: $x^4 \div x^4 = x^{(4-4)} = x^0$

If x is nonzero then: $x^4 \div x^4 = 1$

And so, $\boldsymbol{x^0 = 1}$

In general, $\boldsymbol{x^0 = 1 \text{ (if } x \neq 0)}$

Why do you think x must be nonzero?

Summary of the index laws

Link the equivalent expressions

$x^m \times x^n$	x
$x^m \div x^n$	$x^{(m-n)}$
$(x^m)^n$	$x^n y^n$
$(xy)^n$	x^{mn}
x^0	1 (if $x \neq 0$)
x^1	$x^{(m+n)}$

Complete the calculation using your knowledge of the different exponent laws.

$$(8.5^{14})^{\square} = 8.5^{196}$$

Rice on a Chess Board

It is said that an ancient Indian mathematician invented chess and showed his creation to the ruler of the country.

Press **play** to find out more.

Grotty Greens and Rotten Reds are bacteria types. Both types multiply by splitting in four, but they do this at different rates.

- The Grotty Greens divide every 12 minutes.

- The Rotten Reds divide every 10 minutes.

After one hour, how many bacteria will there be of each type?

After 2 hours, how many times more Rottens are there than Grotties?

Bacteria growth

Grotty Greens:

Time (minutes)	0	12	24	36	48	60
No. of bacteria	1	4	16	64	256	1024

Rotten Reds:

Time (minutes)	0	10	20	30	40	50	60
No. of bacteria	1	4	16	64	256	1024	4096

- In one hour there will be 4^5 = **1024 Grotties**.

  In one hour there will be 4^6 = **4096 Rottens**.
- After 2 hours there will be 4^{10} Grotties and 4^{12} Rottens.

  This means there will be $4^{12} \div 4^{10} = 4^{12-10} = 4^2$ = **16 times** more Rottens than Grotties.

Write a function $f(m)$ for each bacteria type, describing the number of bacteria after m minutes. Plot a graph of these functions using your graphing calculator.

Bacteria growth

We need to write a function $f(m)$ describing the number of bacteria after m minutes.

For the Grotty Greens, after every set of 12 minutes (let's call each set s) there will be 4^s bacteria ($4^0 = 1$ at the start, $4^1 = 4$ after 12 minutes, $4^2 = 16$ after 24 minutes, etc.)

We want the function in terms of the number of minutes, m.

The value of s is the number of minutes, m, divided by 12.

So, the function for the Grotty Greens is: $f(m) = 4^{m/12}$

The function for the Rotten Reds is: $f(m) = 4^{m/10}$