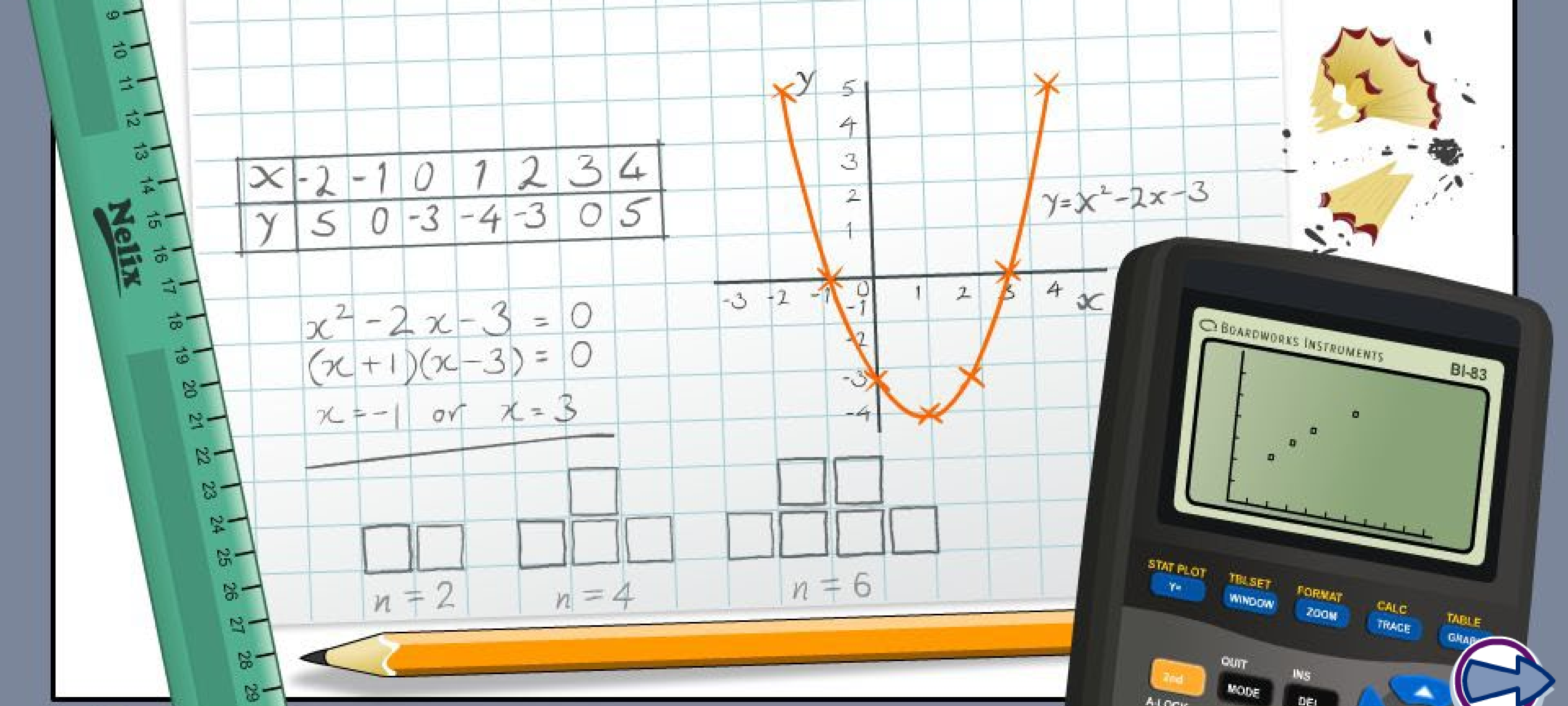
Group and project work



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

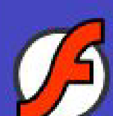
Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**



This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Question: Bill Gates (1)

As of March of 2012 it has been reported that Bill Gates has a net worth of $61 billion. Suppose he sets aside $1 billion for a special education fund. He says that he will donate from this fund in such a way that the balance will decrease in value at the annual rate of ½%, with no further additions to the fund.

Press **start** to see questions a – c.

start

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Press **start** to see questions d – f.

start

Question: standard/vertex form

a) Use the method of completing the square for $y = ax^2 + bx + c$, the standard form of a quadratic function, to create the vertex form: $\boldsymbol{y = a(x - h)^2 + k}$.

b) Use this form to state i) the coordinates of the vertex and ii) the equation of the axis of symmetry, if you are given the parabola's equation in standard form.

Press **play** to see the solution.

Do you think there is a relationship between a person's arm span and their height?

<u>Preliminary work</u>: Measure and record the arm spans and heights of some family members.

<u>In class</u>:

- **Find a partner and measure each other's arm span and height.**
- **Record your results in a table.**
- **Make a scatter plot of the data on graph paper (including your family's measurements.)**
- **Draw a line of best fit for the data and find its equation.**
- **Make a conjecture about the relationship between a person's arm span and their height.**

Questions: arm span

You have already recorded your results, drawn a scatter plot with a line of best fit and made a conjecture about the relationship between a person's arm span and their height.

Now, press **start** to see five questions that will guide you through the analysis of your results.

start

A fireworks flare is shot straight up into the air from the ground with an initial velocity of 160 ft/sec, exploding at its maximum height. The function that models the height of the flare over time is $h(t) = -16t^2 + 160t$. Using your knowledge of quadratic equations and functions, answer the following questions:

a) What is the maximum height that the flare reaches?

b) How many seconds after its launch does it reach this height?

c) If the flare fails to ignite, after how many seconds will it return to the ground?

d) How long does it take to reach a height of 336 feet?

Press **start** to see the solutions to the questions.

start

Question: bike assembly

A small company assembles two types of bicycles: mountain bikes, which require 2 hours to assemble, and road bikes, which require 3 hours. There are at most 96 labor hours available in a day and the manager knows he can assemble a maximum of 30 of each type of bike.

If he makes a profit of $50 on each road bike and $35 on each mountain bike, how many of each type should he assemble to maximize his daily profit? Your tasks:

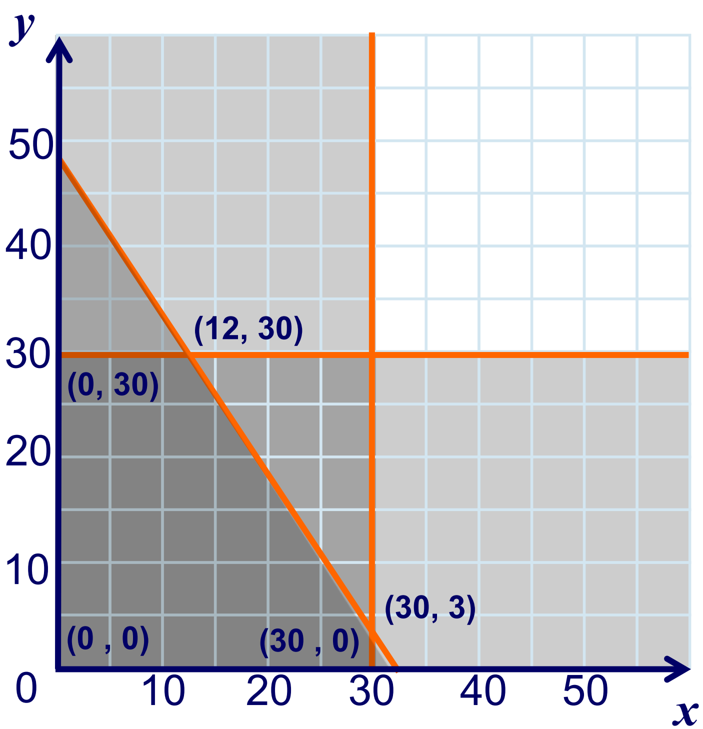
- **Write the system of inequalities (the constraints.)**
- **Graph the inequalities and find the feasibility region. Find the corner points of this region.**
- **State the objective function and use it to help you answer the question.**

Solution: bike assembly

Let x = number of road bikes to assemble

Let y = number of mountain bikes to assemble

Then the constraints are: $0 \leq x \leq 30$, $0 \leq y \leq 30$, $3x + 2y \leq 96$



The objective function is:

$P(x, y) = 50x + 35y$

Corner point	$50x + 35y$
(0, 0)	50(0) + 35(0) = 0
(0, 20)	50(0) + 35(20) = 700
(12, 30)	50(12) + 35(30) = 1650
(30, 3)	50(30) + 35(3) = 1605
(30, 0)	50(30) + 35(0) = 1500

From the table we see that the manager should have **12 road bikes** and **30 mountain bikes** assembled to receive a maximum profit of **$1650**.

The experiment:

Each group needs: 1 plate, 2 cups (1 empty and 1 filled with M&Ms) and a piece of graph paper.

Step 1

Put 4 M&Ms into your empty cup. This is trial 0 and you have 4 M&Ms, which is already recorded for you in the table. Each row forms an ordered pair: (trial #, # of M&Ms).

trial #	total #
0	4
1	
2	
3	
4	
5	
6	
7	
8	

Step 2

Pour these M&Ms onto your plate and count how many fall ‘M’ side up. Take this many M&Ms from your full cup and add them to the pile, then count the total. This is trial 1. Record the result in your table.

Step 3

Return the M&Ms from the plate to your empty cup. Repeat this process until you run out of M&Ms. Record the data in your table at each step (always record the number of M&Ms you have *after* you have added the new ones.)

i) Graph the data points on graph paper. How does your data seem to behave?

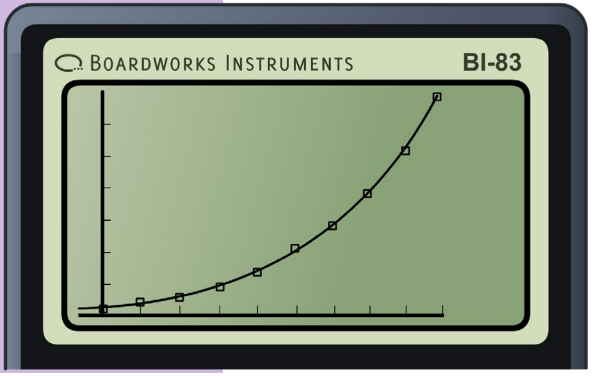
ii) Put your data points into the STAT feature of your calculator to verify your graph.

iii) Choose the type of function you think best fits the data. Use the regression feature in the "STAT CALC" menu to fit a function to it. Run this regression and give the regression equation (to the nearest thousandth).

iv) What is the correlation coefficient? What does it imply about your data? Put the regression line equation in Y1 to see how well it fits your data.

Example results:

trial #	total #
0	4
1	6
2	9
3	13
4	19
5	28
6	41
7	59
8	87



Question: linear or exponential?

x	$f(x)$
1	2
2	5
3	8
4	11
5	14

(i)

x	$f(x)$
1	–4
2	–2
3	–1
4	–0.5
5	–0.25

(ii)

x	$f(x)$
1	–6
2	12
3	–24
4	48
5	–96

(iii)

x	$f(x)$
1	6
2	6.5
3	7
4	7.5
5	8

(iv)

x	$f(x)$
1	2
2	4
3	8
4	16
5	32

(v)

x	$f(x)$
1	–4
2	–6
3	–8
4	–10
5	–12

(vi)

- **Categorize functions i – vi (tabulated above) as being linear or exponential functions.**
- **Write an explanation that could be used to teach your classmates how to decide upon the type.**
- **Write both the recursive and explicit formula that generates the values of the function.**
- **Use your explicit formula and the 'SEQ' feature of the calculator to verify your formula.**

Solution: linear or exponential?

function	type	recursive definition	explicit/function notation
i	linear	$a_1 = 2$, $a_n = a_{n-1} + 3$	$a_n = 3n - 1$ or $f(x) = 3x - 1$
ii	exponential	$a_1 = -4$, $a_n = 0.5a_{n-1}$	$a_n = -4(0.5)^{n-1}$ or $f(x) = = -4(0.5)^{x-1}$
iii	exponential	$a_1 = -6$, $a_n = -2a_{n-1}$	$a_n = -6(-2)^{n-1}$ or $f(x) = -6(-2)^{x-1}$
iv	linear	$a_1 = 6$, $a_n = a_{n-1} + 0.5$	$a_n = 0.5n + 5.5$ or $f(x) = 0.5x + 5.5$
v	exponential	$a_1 = 2$, $a_n = 2a_{n-1}$	$a_n = 2(2)^{n-1}$ or $f(x) = 2(2)^{x-1}$
vi	linear	$a_1 = -4$, $a_n = a_{n-1} - 2$	$a_n = -2n - 2$ or $f(x) = -2x - 2$

In groups, create a linear model for a set of real-world data. This data could come from a textbook, a library book, an experiment, or an internet source. The steps you should follow are shown below.

Part i: describe your data

Part ii: find a line by hand and discuss the fit

Part iii: use your calculator to find the line

Part iv: use your model to make predictions

Part v (optional): display your data

Press each of the buttons on the left to see more details about each part of the task.

Project rubric and discussion points

Part i:	• There is a description of the data set used. • The write-up is typed, grammatically correct and error-free.
Part ii:	• The scatter plot is done using an appropriate scale. • The by-hand linear model is calculated correctly. • The linear model is drawn neatly on the scatter plot. • There is some discussion of the fit.
Part iii:	• The calculator regression is correct and includes the correlation coefficient correct to 3 decimal places. • The regression line is accurate and in a different color. • There is some discussion comparing the calculator and regression to the by-hand model. • There is a discussion of r.
Part iv:	• There is an x-value prediction and related discussion. • There is a prediction of a y-value and related discussion.

Gasoline prices vary greatly, even within a city.

- **Survey the gasoline prices for one particular grade of gas at as many stations as you can over the weekend.**
- **Analyze the data you collected using different measures of central tendency.**
- **Create a visual presentation of your results and write a report. Refer to the sampling method you used and discuss whether or not your sampling could have been biased.**

Include one or more of the graphs that you have studied relating to data analysis and as many vocabulary words from this unit as possible.

Project rubric and discussion points

- The write-up is mathematically correct and presented without grammatical errors.
- The sampling method/population is described and the possibility of biased data is discussed.
- The measures of central tendency are shown and discussed (including an explanation of which measure is most accurate for the data.)
- The measures of dispersion are noted:
  - the range
  - the mean absolute deviation
  - the quartile values (upper and lower), interquartile range
  - any outliers, including a discussion as to why a particular gas station’s price may be an outlier.
- An appropriate visual display for the data has been chosen.

You have learned the shortcut to square a binomial:
$(a + b)^2 = a^2 + 2ab + b^2$.
For example, $(x + 1)^2 = x^2 + 2x + 1$
A French mathematician and philosopher called **Blaise Pascal** developed a process for finding $(a + b)^n$ for a given integer, n.
For example, how could we quickly get $(x + 1)^5$?

Research **Pascal’s triangle** and prepare a short paper discussing how it is used to raise a binomial to a power. Be prepared to explain the process and demonstrate its use for several values of n.

Press **start** to read about Pascal's triangle and check your results.

start