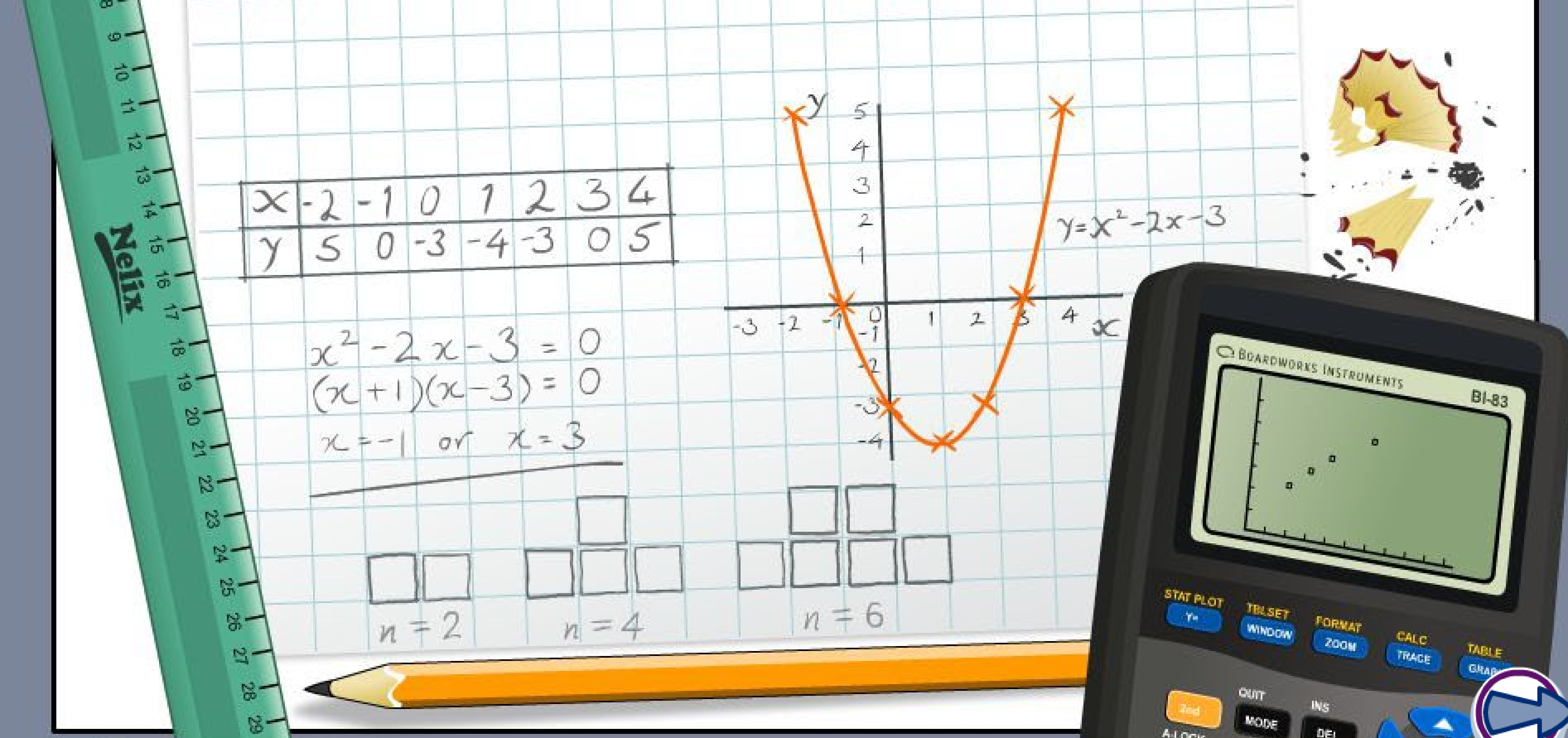
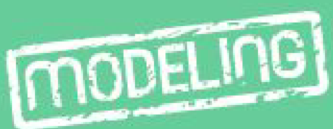
Linear graphs



Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



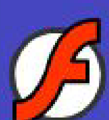
Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

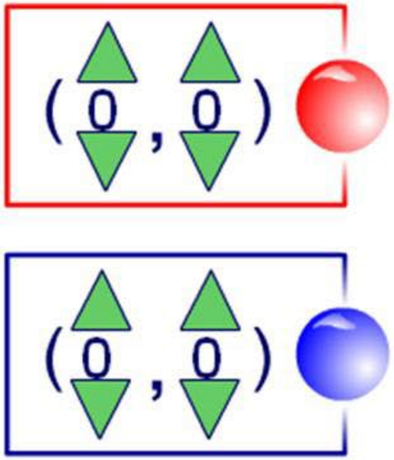


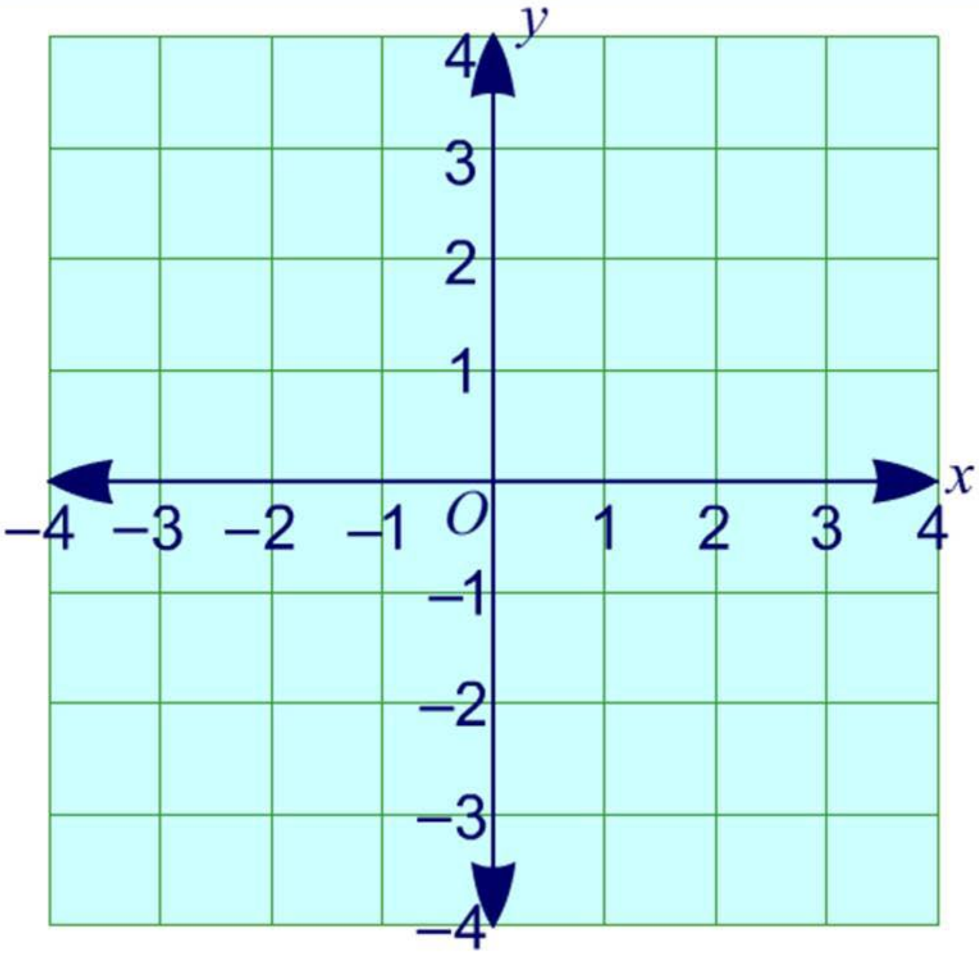
This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Don't connect 3!

Get into two teams.
Take it in turns to plot a point on the graph.
You lose if you plot 3 points in a row!

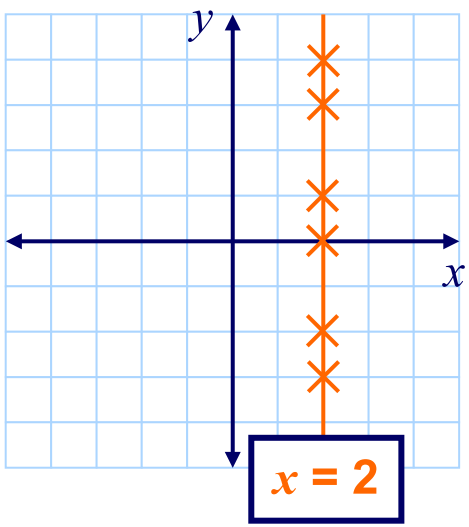




What do these coordinate pairs have in common: (2, 3), (2, 1), (2, –2), (2, 4), (2, 0) and (2, –3)?

The x-coordinate in each pair is equal to 2.

Look what happens when these points are plotted on a graph.



All of the points lie on a straight line parallel to the y-axis.

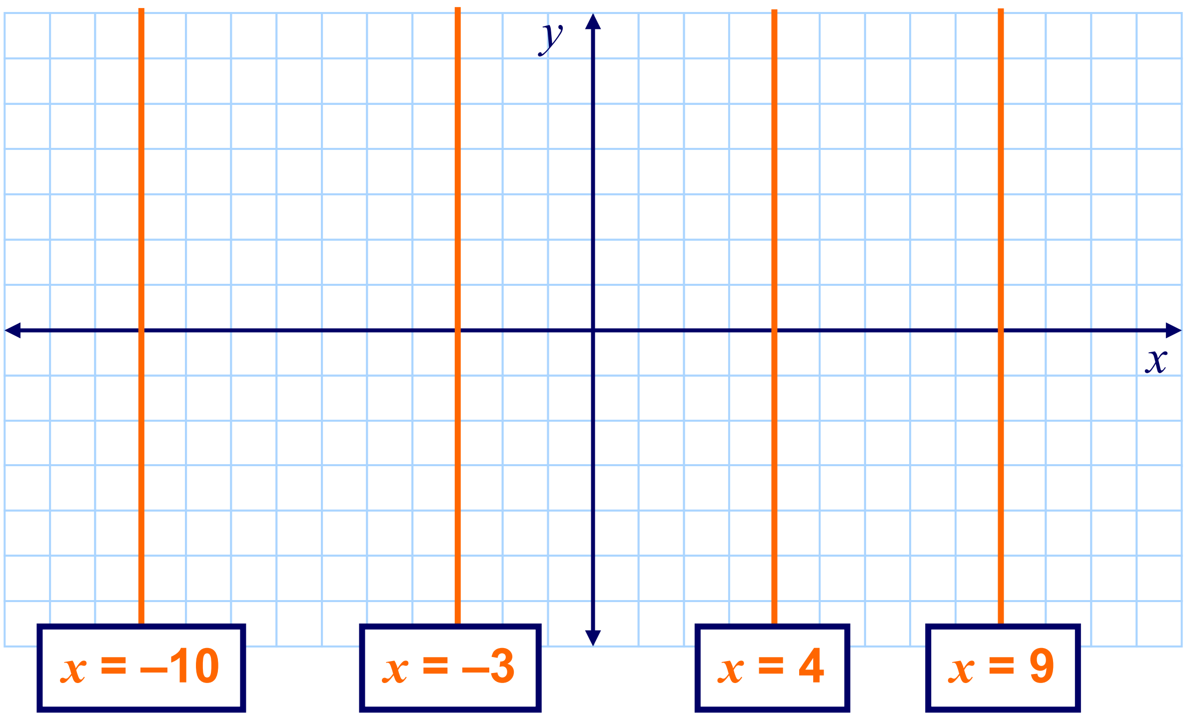
Name five other points that will lie on this line.

This line is called $x = 2$.

All graphs of the form

$$x = c,$$

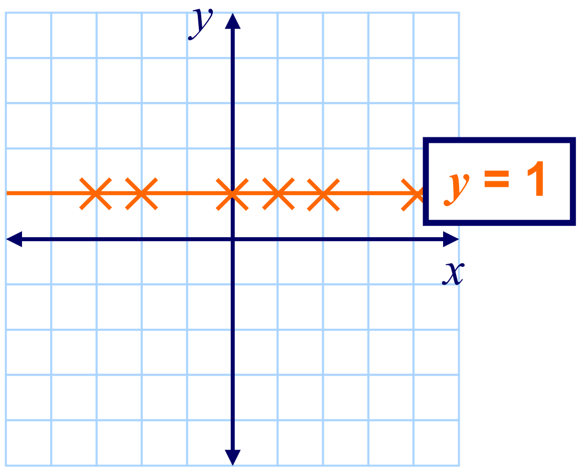
where c is any real number, will be parallel to the y-axis and will intersect the x-axis at the point **(c, 0)**.



What do these coordinate pairs have in common?
(0, 1), (4, 1), (–2, 1), (2, 1), (1, 1) and (–3, 1)?

The y-coordinate in each pair is equal to 1.

Look at what happens when these points are plotted on a graph.



All of the points lie on a straight line parallel to the x-axis.

Name five other points that will lie on this line.

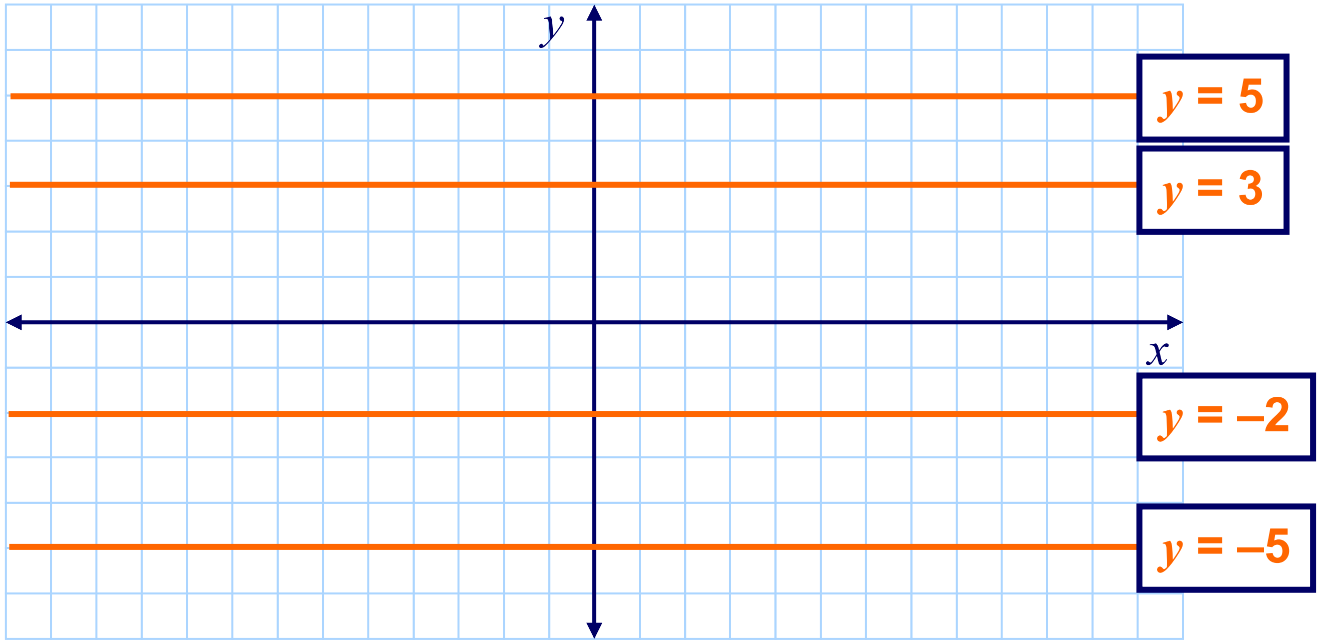
This line is called $y = 1$.

Graphs parallel to the x-axis

All graphs of the form

$$y = c,$$

where c is any real number, will be parallel to the x-axis and will intersect the y-axis at the point **(0, c)**.



Drawing graphs of linear functions

The x-coordinate and the y-coordinate in a coordinate pair can be linked by a function.

What do these coordinate pairs have in common?
(1, –1), (4, 2), (–2, –4), (0, –2), (–1, –3) and (3.5, 1.5)?

In each pair, the y-coordinate is 2 less than the x-coordinate.

These coordinates are linked by the function:

$$y = x - 2$$

We can draw a graph of the function $y = x - 2$ by plotting points that obey this function.

Drawing graphs of linear functions

Given a function, we can find coordinate points that obey the function by constructing a **table of values**.

Suppose we want to plot points that obey the function

$$y = 2x + 5$$

We can use a table as follows:

x	–3	–2	–1	0	1	2	3
$y = 2x + 5$	–1	1	3	5	7	9	11
	(–3, –1)	(–2, 1)	(–1, 3)	(0, 5)	(1, 7)	(2, 9)	(3, 11)

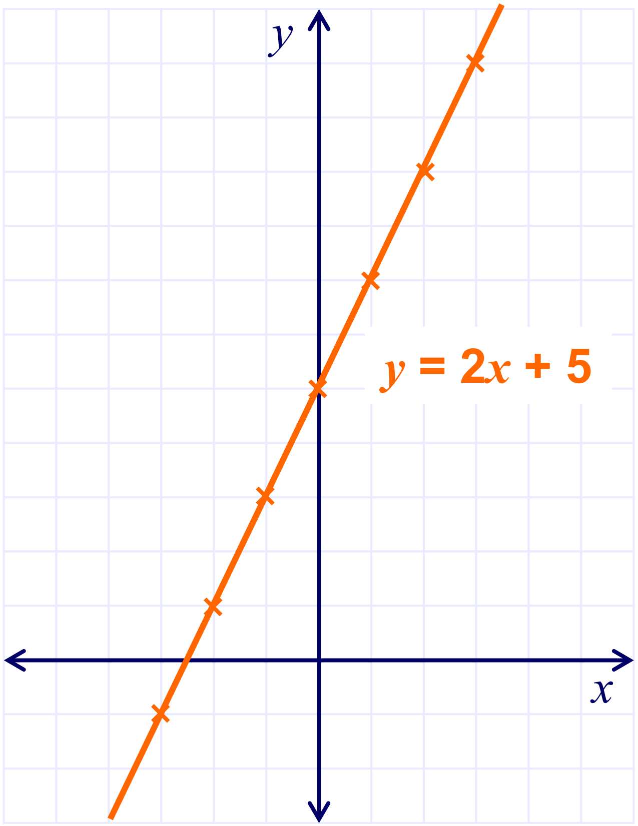
Drawing graphs of linear functions

For example,

to draw a graph of $y = 2x + 5$:

1) Complete a table of values:

x	–3	–2	–1	0	1	2	3
$y = 2x + 5$	–1	1	3	5	7	9	11



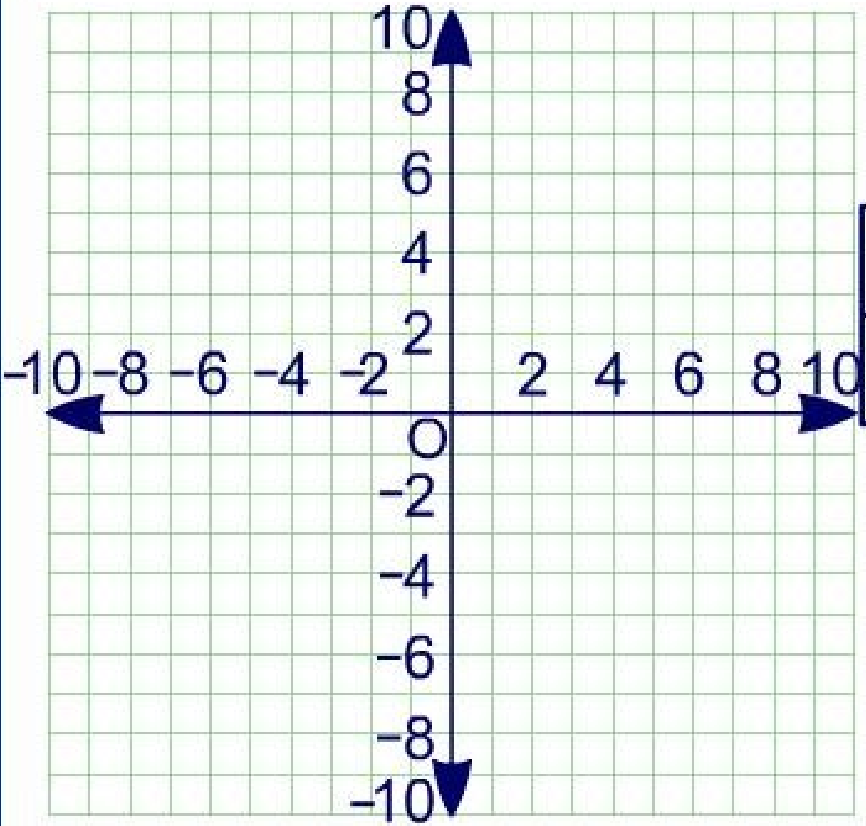
2) Plot the points on a coordinate grid.

3) Draw a line through the points.

4) Label the line.

5) Check that other points on the line fit the rule.

Plotting graphs of linear functions



$y = 5x + 1$

x	–3	–2	–1	0	1	2	3
y							

show line

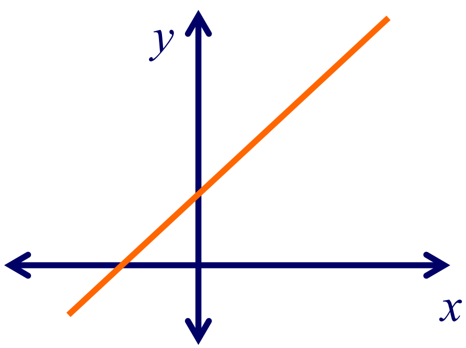
show crosshair

Slopes of straight-line graphs

The **slope** of a line is a measure of how steep the line is.

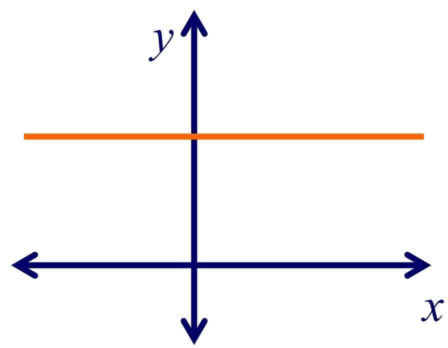
The slope of a line can be positive, negative or zero.

an upwards slope



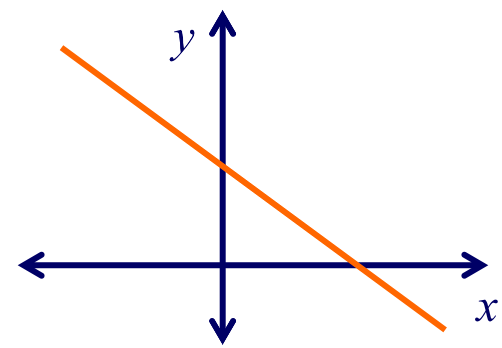
positive slope

a horizontal line



zero slope

a downwards slope



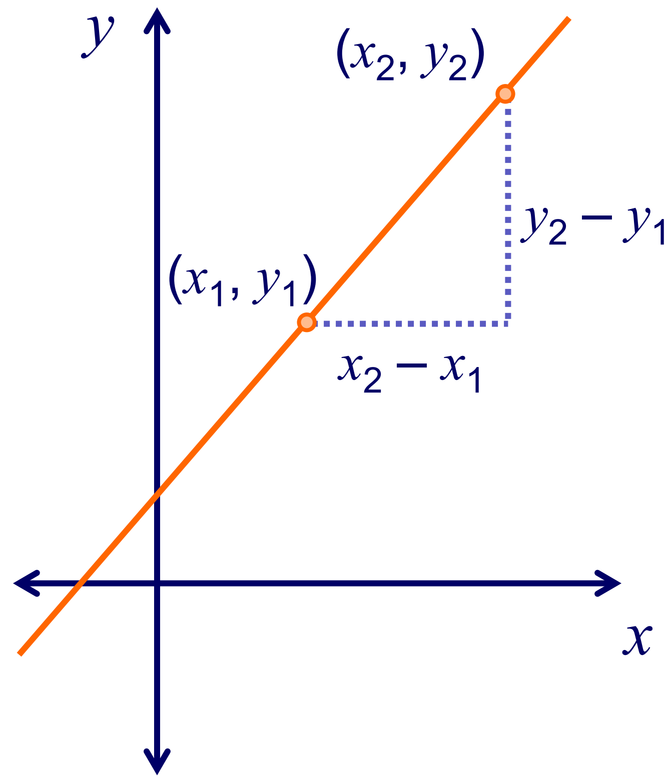
negative slope

If a line is vertical, its slope cannot be specified.
We often say the slope is “undefined”.

Calculating the slope

If we are given any two points (x_1, y_1) and (x_2, y_2) on a line we can calculate the slope of the line as follows:

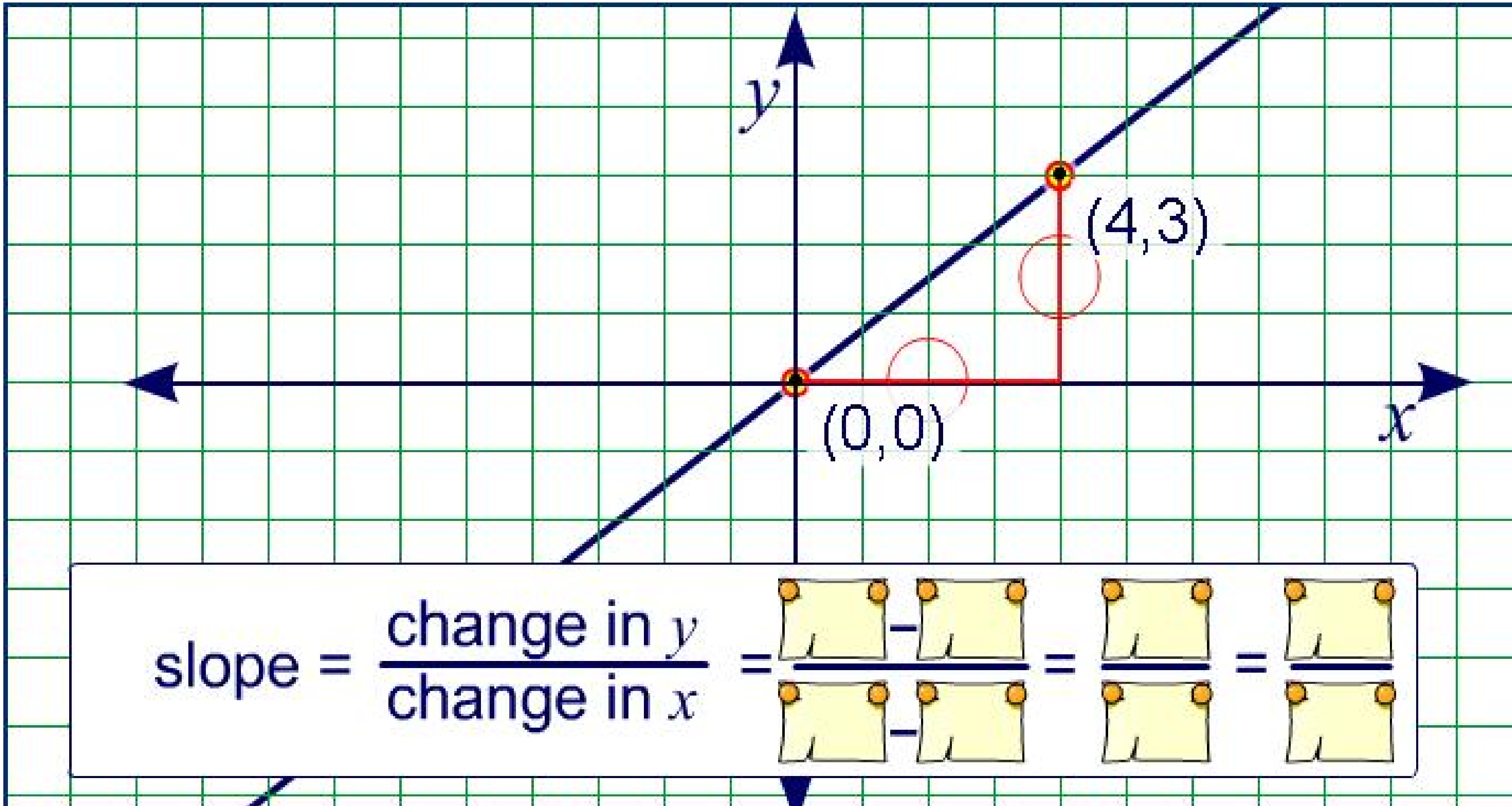
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\textbf{rise}}{\textbf{run}}$$



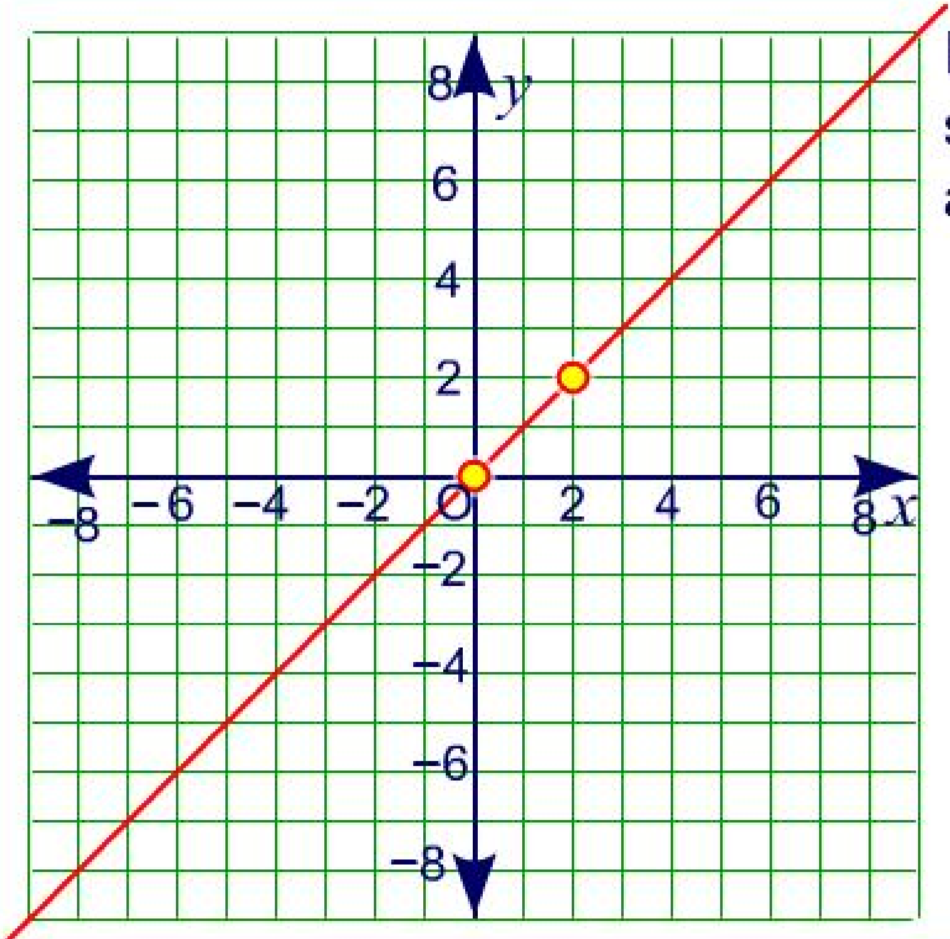
To help you find this more easily, you can draw a right triangle between the two points on the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Calculating slopes



$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} =$$



Investigate how adjusting the slope (m) and y-intercept (b) affects the graph.

$y = x$

$b = 0$

slope = $\frac{5}{5}$

The general equation of any straight line is:

$$y = mx + b$$

This is called the **slope-intercept form** of a straight line equation.

The value of m tells us the **slope** of the line.

The value of b tells us where the line crosses the y-axis.

This is called the **y-intercept** and it has the coordinates $(0, b)$.

For example, the line $y = 3x + 4$ has a slope of **3** and crosses the y-axis at the point (0, **4**).

Link the equation with its slope and y-intercept.

equation	slope	y-intercept
$y = 5x - 3$	1	(0, –3)
$y = \frac{x}{2} - 5$	–3	(0, 0)
$y = 2 - 3x$	–2	(0, –5)
$y = x$	5	$(0, \frac{1}{2})$
$y = -2x + \frac{1}{2}$	$\frac{1}{2}$	(0, 2)

Sometimes the equation of a straight line graph is not given in slope-intercept form, $y = mx + b$.

The equation of a straight line is $2y + x = 4$.
Find the slope and the y-intercept of the line.

Rearrange the equation by performing the same operations on both sides.

$$2y + x = 4$$

subtract x from both sides: $$2y = -x + 4$$

divide both sides by 2: $$y = \frac{-x + 4}{2}$$

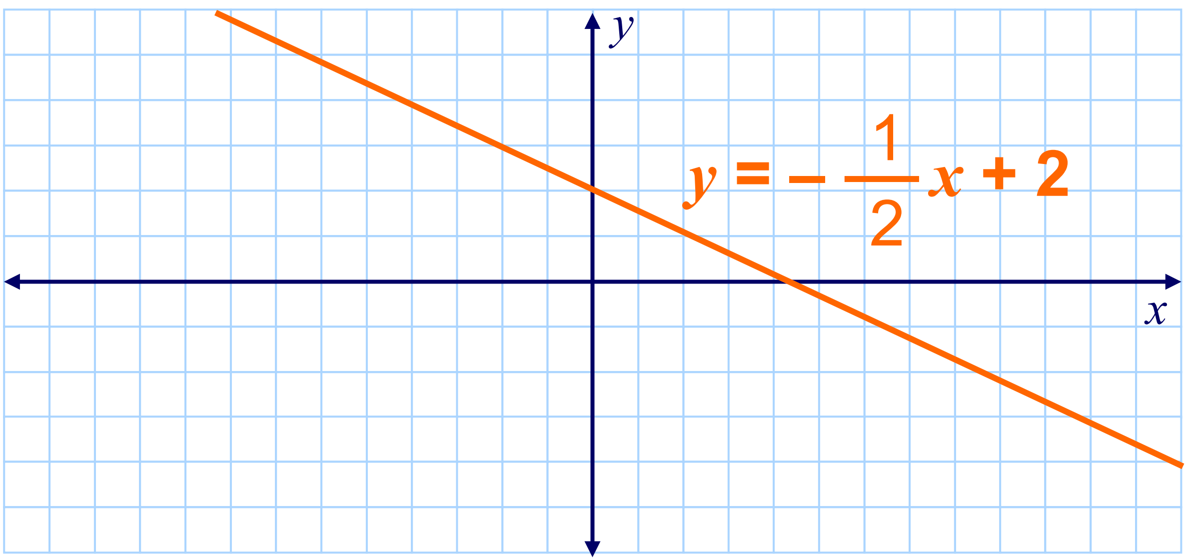
$$y = -\frac{1}{2}x + 2$$

Once the equation is in slope-intercept form, $y = mx + b$, we can determine the value of the slope and the y-intercept.

$$y = -\frac{1}{2}x + 2$$

$m = -\frac{1}{2}$ so the slope of the line is $\mathbf{-\frac{1}{2}}$.

$b = 2$ and so the y-intercept is **(0, 2)**.



Substituting values into equations

A line with the equation $y = mx + 5$ passes through the point (3, 11). What is the value of m?

To solve this problem, we can substitute $x = 3$ and $y = 11$ into the equation $y = mx + 5$.

This gives us: $11 = 3m + 5$

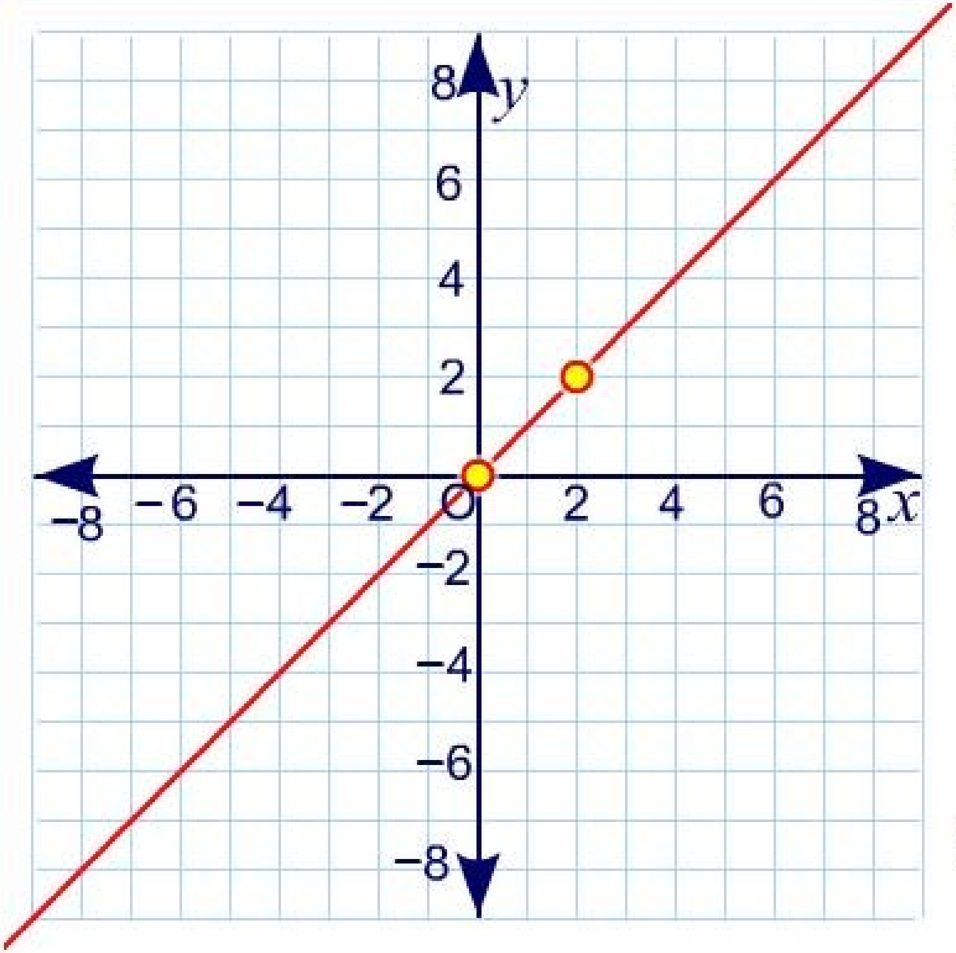
subtract 5 from both sides: $6 = 3m$

divide both sides by 3: $2 = m$

$$m = 2$$

The equation of this line is therefore $y = 2x + 5$.

What is the equation of the line?



Practice finding the equation of the line. Adjust the line by dragging the yellow dots.

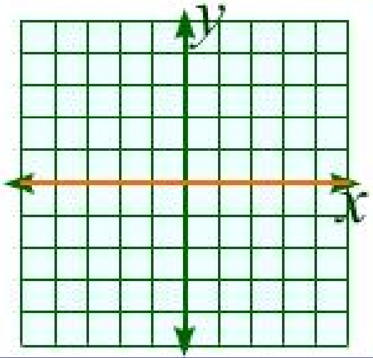
y-intercept:

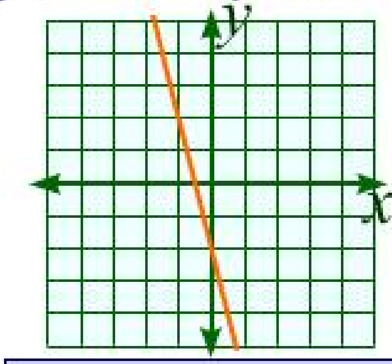
slope:

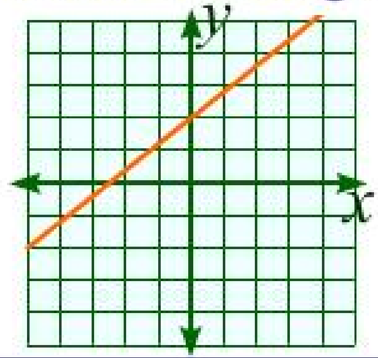
equation:

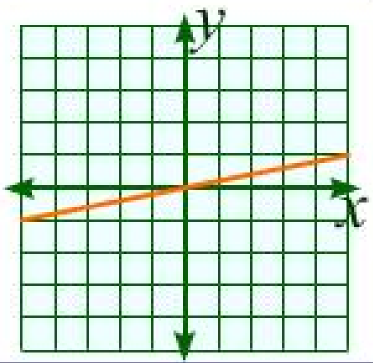
Match the equations to the graphs

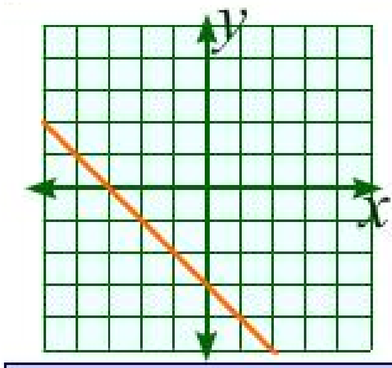
Drag each equation to the correct graph.

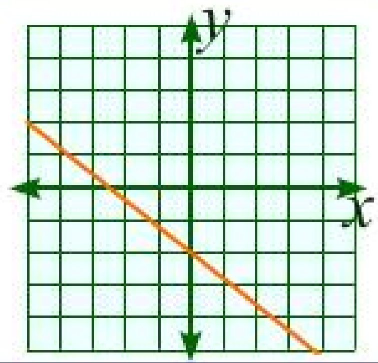






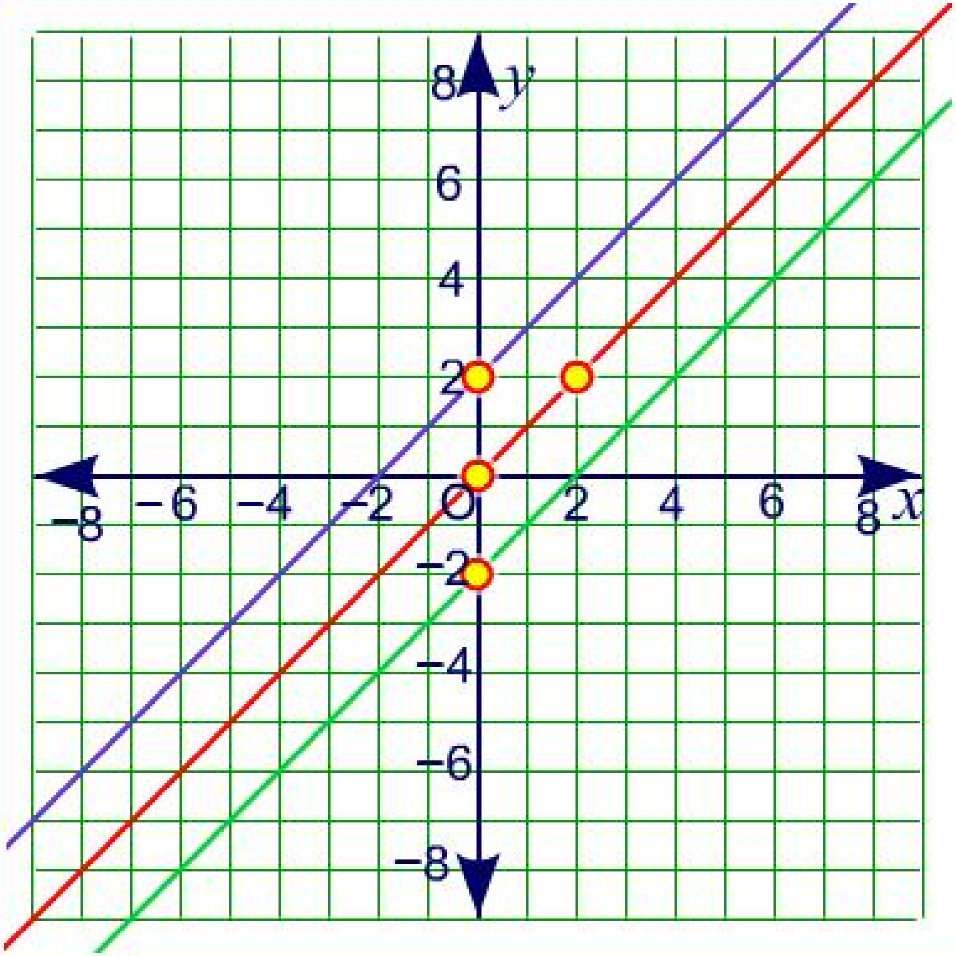






- $y = x - 3$
- $y = \frac{4}{5}x + 2$
- $y = 0$
- $y - \frac{1}{5}x = 0$
- $y + \frac{4}{5}x = -2$
- $y = -4x - 2$

Investigating parallel lines



$y = x$

$y = x - 2$

$y = x + 2$

If two lines have the same slope, they are parallel.

Show that the lines $2y + 6x = 1$ and $y = -3x + 4$ are parallel.

We can show this by rearranging the first equation so that it is in slope-intercept form, $y = mx + b$.

$$2y + 6x = 1$$

subtract $6x$ from both sides:

$$2y = -6x + 1$$

divide both sides by 2:

$$y = \frac{-6x + 1}{2}$$

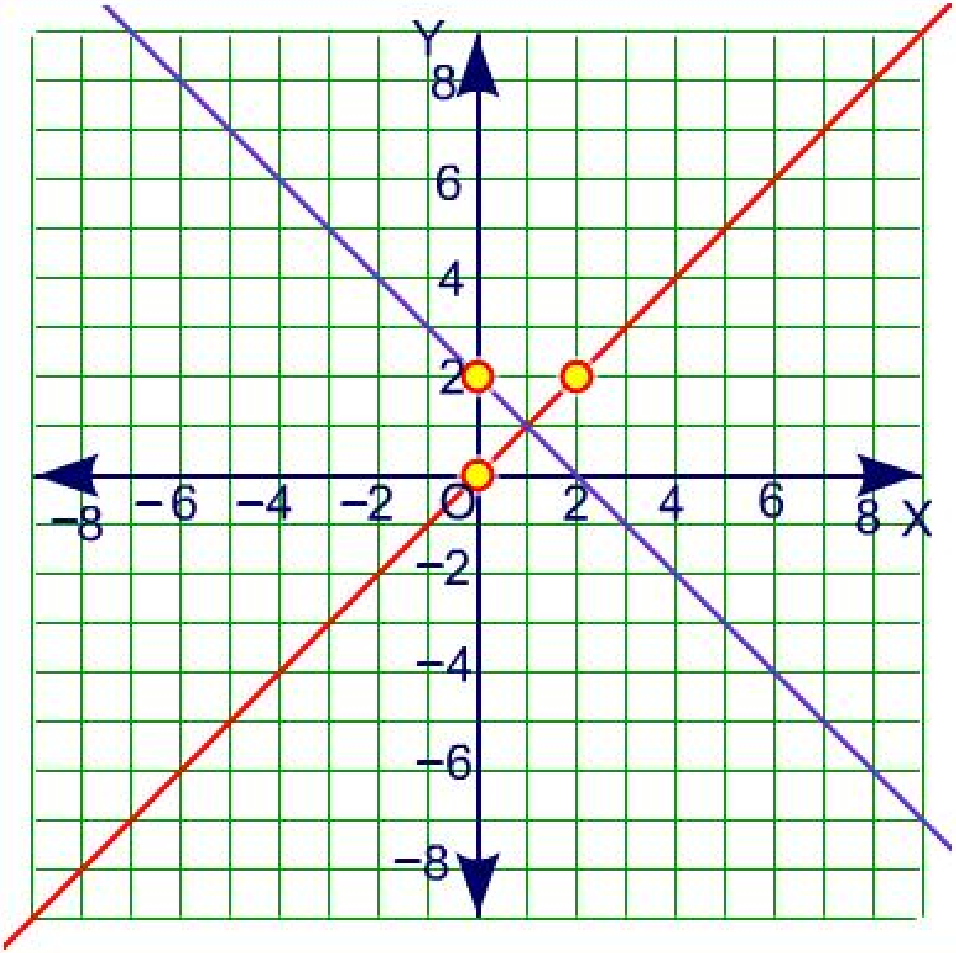
$$y = -3x + \tfrac{1}{2}$$

The slope m is -3 for both lines, so they are parallel.

Matching parallel lines

$y = \frac{-1}{5}x - 4$	$y = \frac{-1}{5}x - 8$
$y = \frac{8}{3}x + 6$	$y = \frac{8}{3}x + 1$
$y = \frac{-1}{2}x + 4$	$y = \frac{-1}{2}x + 3$
$y = \frac{-1}{2}x + 10$	$y = \frac{-1}{2}x - 1$
$y = \frac{-1}{6}x - 5$	$y = \frac{-1}{6}x - 4$

1 2



$y = x$

$y = -x + 2$

Perpendicular lines

If the slopes of two lines have a product of –1, then they are perpendicular.

In general,

> if the slope of a line is m, then the slope of the line perpendicular to it is $-\frac{1}{m}$.

Write down the equation of the line that is perpendicular to $y = -4x + 3$ and passes through the point (0, –5).

The slope of the line $y = -4x + 3$ is –4.

The slope of the line perpendicular to it is therefore $\frac{1}{4}$.

The equation of the line with slope $\frac{1}{4}$ and y-intercept –5 is:

$$y = \frac{1}{4}x - 5$$

Matching perpendicular lines

$y = -x - 5$	$y = x - 1$
$y = \frac{6}{7}x + 6$	$y = \frac{-7}{6}x - 3$
$y = -2x - 5$	$y = \frac{1}{2}x + 10$
$y = -x + 4$	$y = x - 6$
$y = x - 4$	$y = -x - 2$

Investigate the cost of these music download services:

Z-Tunes:
No monthly charge.
$0.99 per song

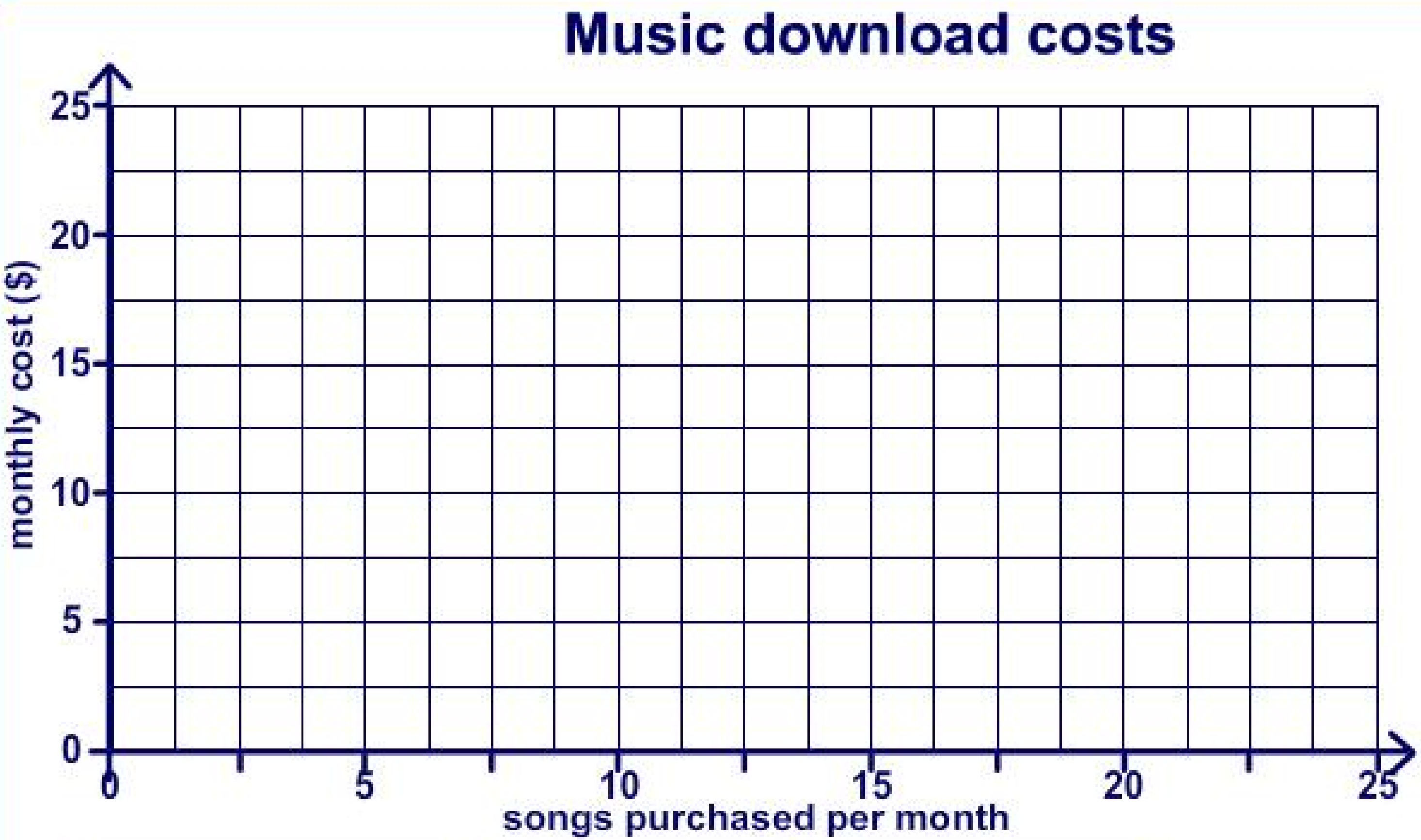
Yes Trax
$9.99 per month
$0.49 per song

hi-notes:
$12.95 per month,
$0.79 per song

MP3 House:
$19.99 per month,
no charge per song

Write the monthly cost of each service as a function of the number of songs downloaded. Compare the services with a graph and a table.

Graphing the costs



Show equations | Show lines | Show table

Customs trap

A customs officer suspects that two boats will meet to transfer smuggled goods.

Using radar he writes down their coordinates at 5pm and 6pm.

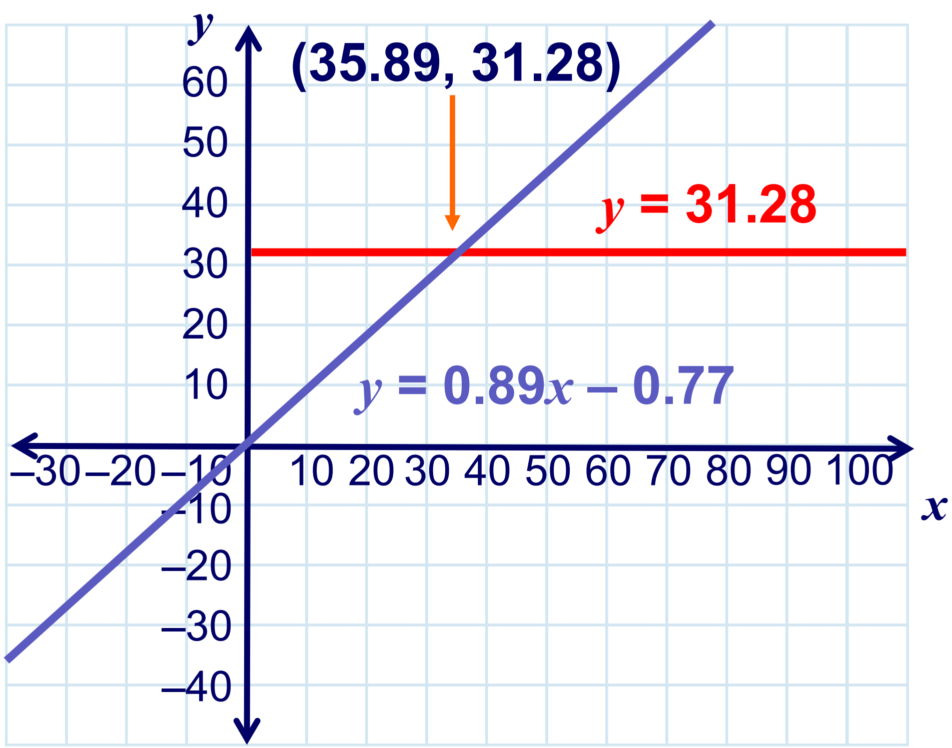
The first boat is at (–32.64, –29.82) at 5pm, and at (–10.05, –9.64) at 6pm.

The second boat is at (105.64, 30.98) at 5pm and (77.18, 31.06) at 6pm.

Sketch their paths on a graph, and write equations for these lines.

Where should the officer expect the boats to meet?

The equations for the two boats' paths are:
First boat: $y = 0.89x - 0.77$
Second boat: $y = 31.28$



Here is a graph of their paths.

Their meeting occurs at the intersection of the lines.

At approximately what time should the officer expect the boats to meet?