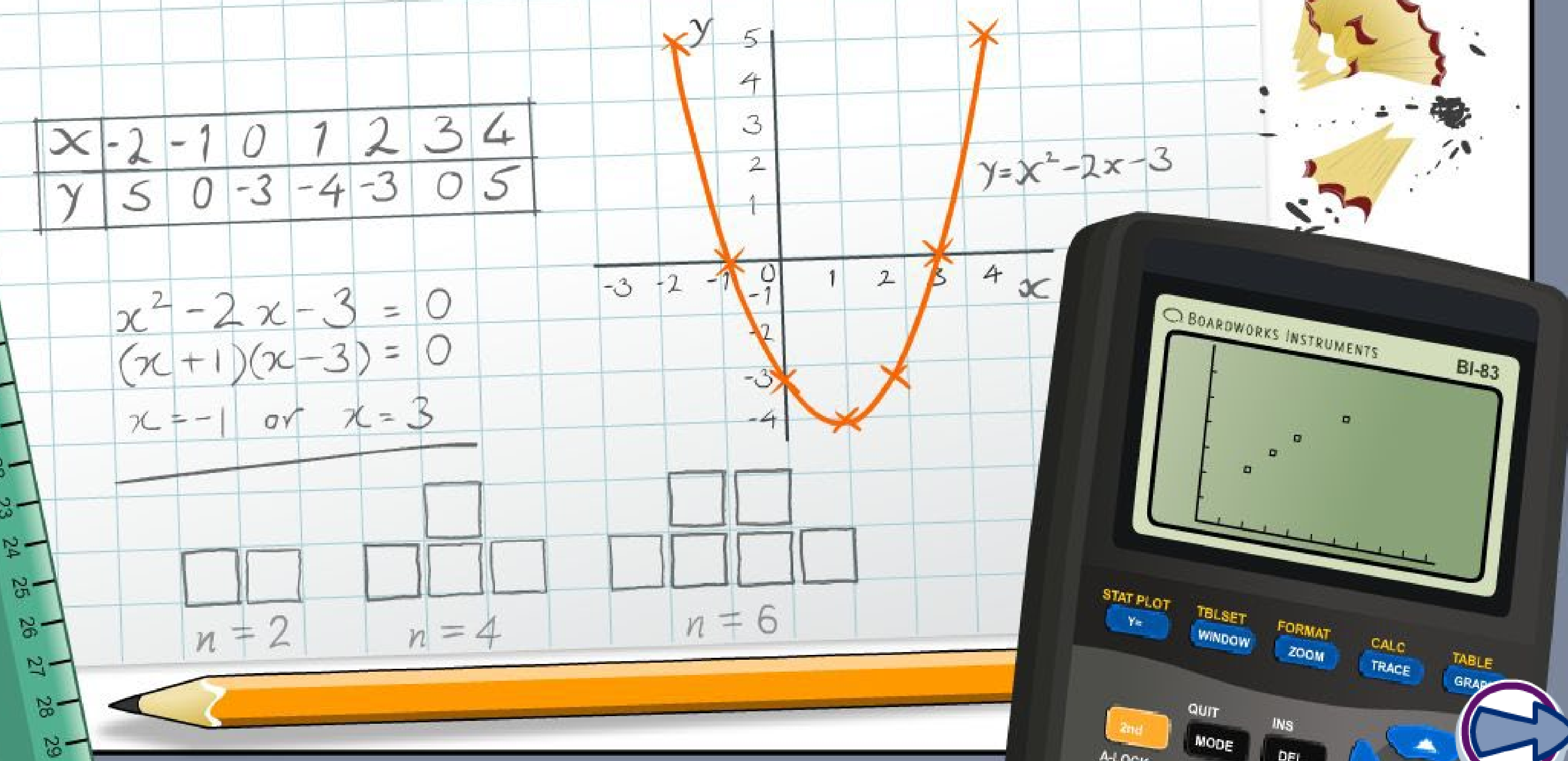
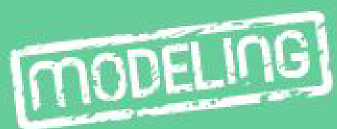
# Operations with polynomials



# Information

## Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.




Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

# Polynomials

A **polynomial in $x$** is an expression of the form

$$ax^n + bx^{n-1} + cx^{n-2} + \ldots + px^2 + qx + r$$

where $a$, $b$, $c$, … are constant coefficients and $n$ is a nonnegative integer.

$a$ is called the **leading coefficient**.

Examples of polynomials include:

$3x^7 + 4x^3 - x + 8$

$x^{11} - 2x^8 + 9x$

and

$5 + 3x^2 - 2x^3$.

# Polynomials

The **degree**, or **order**, of a polynomial is given by the highest power of the variable.

- A polynomial of degree 1 is called **linear** and has the general form $ax + b$.
- A polynomial of degree 2 is called **quadratic** and has the general form $ax^2 + bx + c$.
- A polynomial of degree 3 is called **cubic** and has the general form $ax^3 + bx^2 + cx + d$.
- A polynomial of degree 4 is called **quartic** and has the general form $ax^4 + bx^3 + cx^2 + dx + e$.

# Using function notation

Polynomials are often expressed using function notation.

For example, consider the polynomial function:

$$f(x) = 2x^2 - 7$$

We can use this notation to substitute given values of $x$.

**Find $f(x)$ when a) $x = -2$**
**b) $x = t + 1$**

a) $f(-2) = 2(-2)^2 - 7$

$= 8 - 7$

$\mathbf{= 1}$

b) $f(t + 1) = 2(t + 1)^2 - 7$

$= 2(t^2 + 2t + 1) - 7$

$= 2t^2 + 4t + 2 - 7$

$\mathbf{= 2t^2 + 4t - 5}$

When two or more polynomials are added, subtracted or multiplied, the result is another polynomial.

Polynomials are added and subtracted by **combining like terms**.

**For example:** $f(x) = 2x^2 - 5x + 4$ **and** $g(x) = 2x - 4$

**Find:** a) $f(x) + g(x)$
b) $f(x) - g(x)$

a) $f(x) + g(x)$

$= 2x^2 - 5x + 4 + 2x - 4$

$= \mathbf{2x^2 - 3x}$

b) $f(x) - g(x)$

$= 2x^2 - 5x + 4 - (2x - 4)$

$= 2x^2 - 5x + 4 - 2x + 4$

$= \mathbf{2x^2 - 7x + 8}$

# Multiplying polynomials

When two polynomials are multiplied together, every term in the first polynomial must by distributed to every term in the second polynomial.

The distributive property is used to rewrite the expression without parentheses.

For example: $f(x) = 3x^2 - 2$ and $g(x) = x^2 + 5x - 1$

$$f(x)\,g(x) = (3x^2 - 2)(x^2 + 5x - 1)$$

$$= 3x^4 + 15x^3 - 3x^2 - 2x^2 - 10x + 2$$

$$\mathbf{= 3x^4 + 15x^3 - 5x^2 - 10x + 2}$$

## Performing arithmetic operations on polynomials

For each of the following pairs of polynomials, calculate:

a) $f(x) + g(x)$ b) $f(x) - g(x)$ c) $f(x) \times g(x)$

| | |
|---|---|
| 1. | $f(x) = 5x + 7$, $g(x) = 2x + 9$ |
| 2. | $f(x) = x^2 + 4x - 2$, $g(x) = 3x + 2$ |
| 3. | $f(x) = 2x^2 - 3x + 1$, $g(x) = -x^2 + 6x + 3$ |