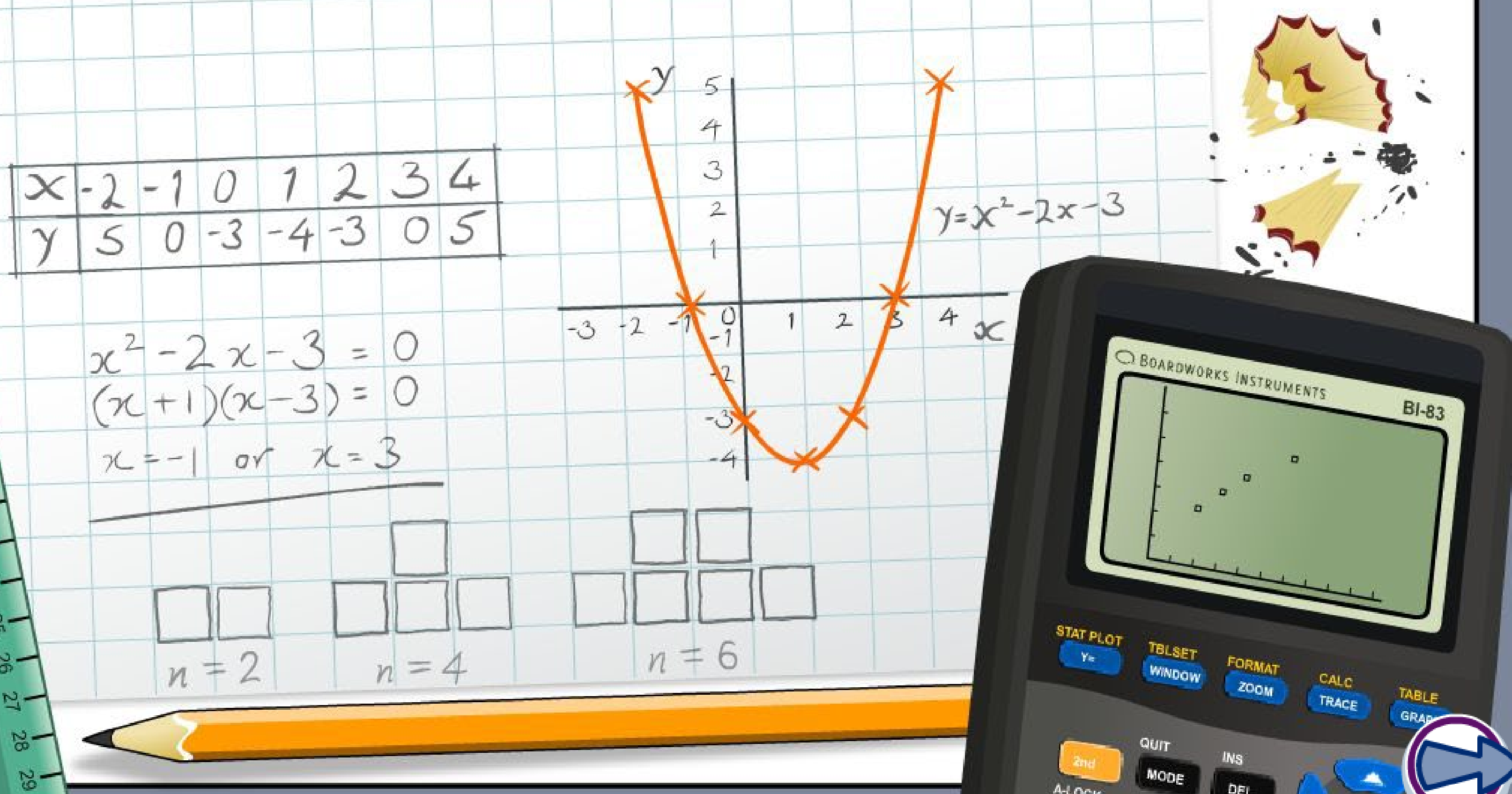
Piecewise-defined functions



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Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

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Piecewise-defined functions

We define a **piecewise function** with a minimum of two functions, each applying to a different part of the **domain**.

For example,

What is $|x|$ (the absolute value of x) when x is positive? How about when x is negative?

The definition of $|x|$ is different for different values of x:

$$|x| = x \text{ if } x \geq 0$$

$$\text{but } |x| = -x \text{ if } x < 0$$

We can define the **absolute value function** as a piecewise function like this:

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

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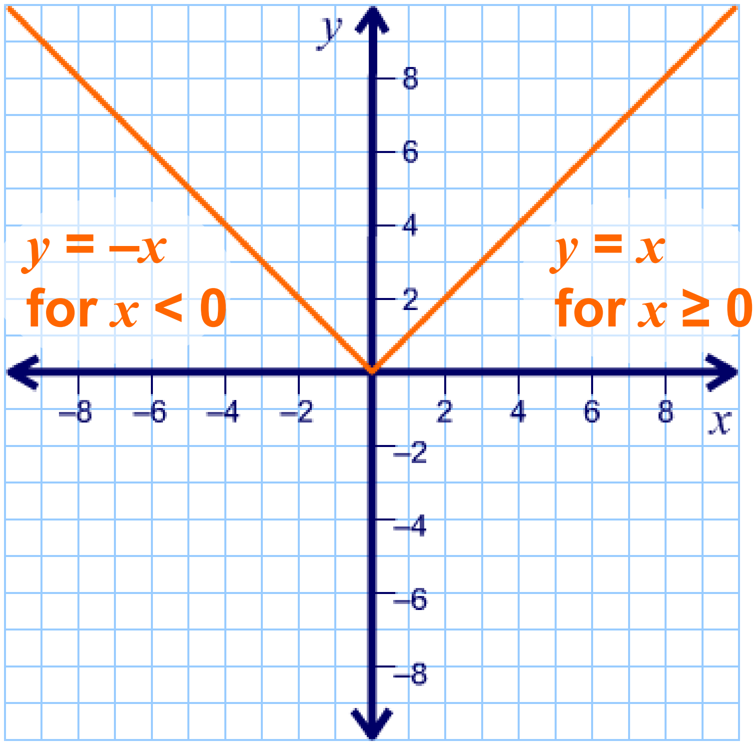
The absolute value function is defined as:

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

We can graph this in its two parts:



The domain for $y = x$ is $x \geq 0$.

The domain for $y = -x$ is $x < 0$.

What is the domain of $f(x)$?

The domain of $f(x)$ is $(-\infty, \infty)$

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Graphing a piecewise function

$$f(x) = \begin{cases} -x - 2, & \text{if } x < -1 \\ 2x + 1, & \text{if } x \geq -1 \end{cases}$$

How would you graph this piecewise function?

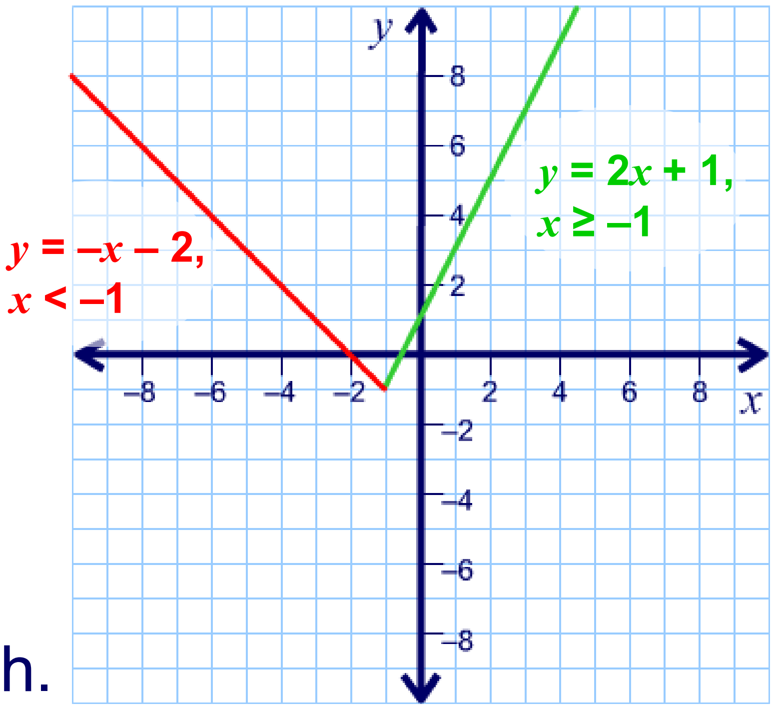
Identify the first equation we need to graph and its domain:

$y = -x - 2$ where $x < -1$

Identify the second equation we need to graph and its domain:

$y = 2x + 1$ where $x \geq -1$

Use graph paper to sketch the graph.



This is an example of a "continuous" piecewise function. What do you think this means?

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Circles at endpoints

When graphing a piecewise function, we must examine the **critical points** (here, the endpoints) of each domain to see whether or not the function is defined at these points.

- If the endpoint is defined, draw a closed circle.
- If the endpoint is not defined, draw an open circle.

A **continuous** function can be thought of as one that can be drawn without lifting your pencil off of the paper, i.e. it does not jump, and all rays or segments join together with no gaps.

If two segments join at a point that is undefined by both domains, then there is a **hole**.

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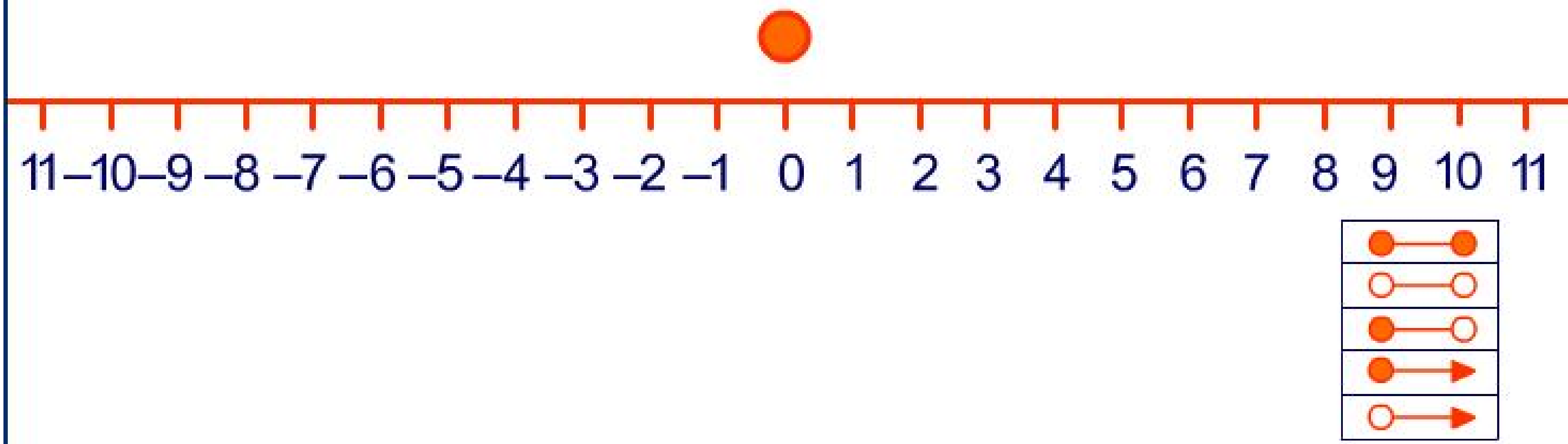
Recap: inequalities on number lines

Represent this inequality on the number line:

$-7 < x < 2$



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Graphing practice 1

When graphing a piecewise-defined function, ask yourself: 'What are the critical points to examine? Will there be any holes in the completed graph?'

Press **start** to see an example question.

start

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Graphing practice 2

When graphing a piecewise-defined function, ask yourself: 'What are the critical points to examine? Will there be any holes in the completed graph?'

Press **start** to see an example question.

start

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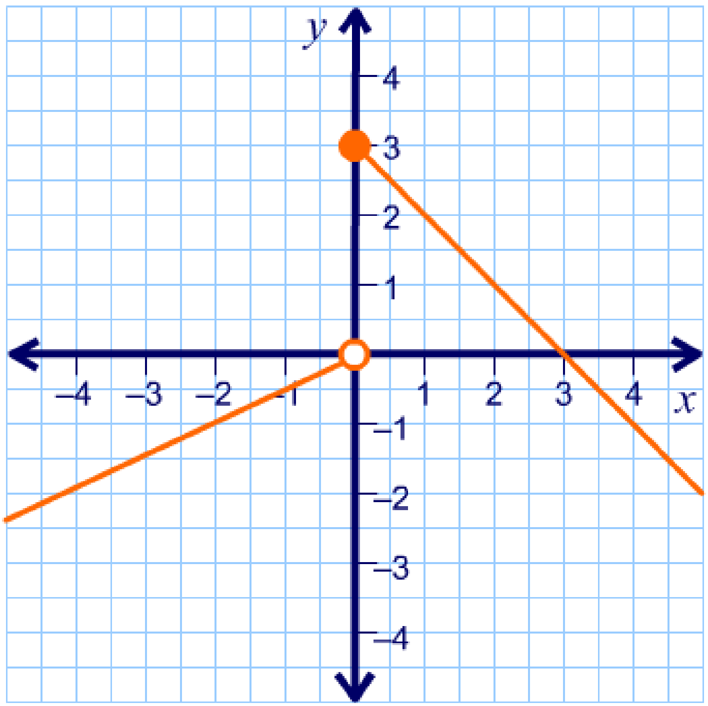
Write the piecewise function

Write the piecewise function for the graph shown.



There are two parts (or "rays") to this piecewise function.

The open circle at (0, 0) means that the domain for the left ray is $x < 0$. Its equation is $y = \frac{1}{2}x$.

The closed circle at (0, 3) means that the domain for the right ray is $x \geq 0$. Its equation is $-x + 3$.

The piecewise function is:

$$f(x) = \begin{cases} \frac{1}{2}x, & \text{if } x < 0 \\ -x + 3, & \text{if } x \geq 0 \end{cases}$$

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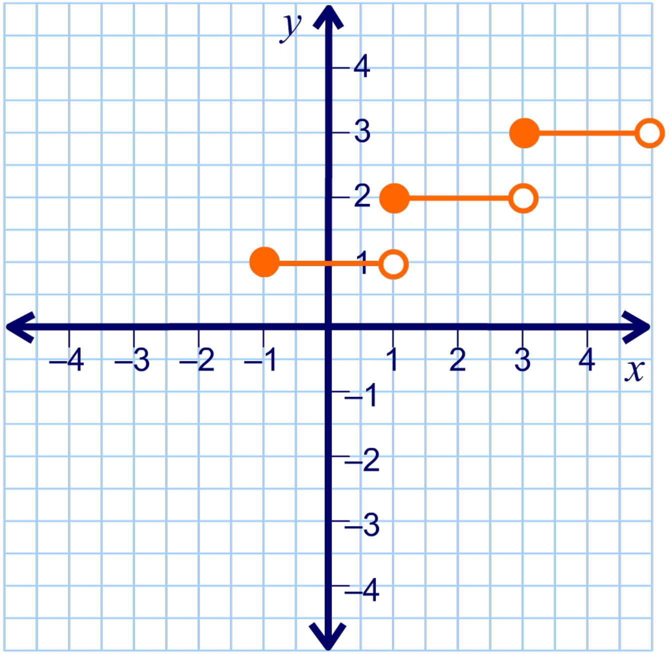
A **step function** is piecewise function made up of horizontal components that jump up or down at certain values.

We call it a step function because it resembles a set of stairs. It is made up of horizontal segments where, over specific intervals, the y-value of the function remains constant.



Write the function in the graph in the form of a piecewise-defined function.

$$f(x) = \begin{cases} 1, & \text{if } -1 \leq x < 1 \\ 2, & \text{if } 1 \leq x < 3 \\ 3, & \text{if } 3 \leq x < 5 \end{cases}$$

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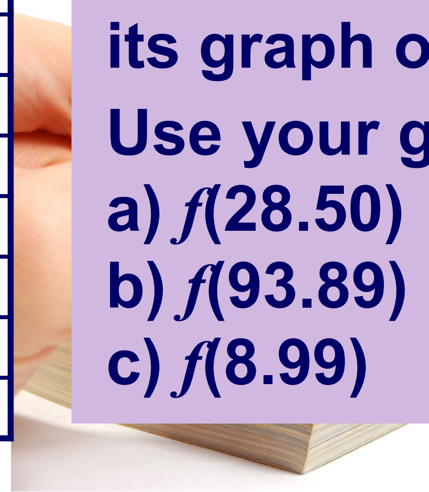
Step function application

Where can we use a step function in real life?
Can you think of an instance where the cost of an item is constant over a specific interval?

Below is a table containing shipping and handling fees for a catalog company.

merchandise total	shipping and handling
Under $14.99	$5.99
$15.00 to $29.99	$7.99
$30.00 to $44.99	$8.99
$45.00 to $59.99	$9.99
$60.00 to $74.99	$11.99
$75.00 to $89.99	$12.99
$90.00 to $119.99	$14.99
$120.00 and over	$17.99



Write a step function to model this data and draw its graph on graph paper.

Use your graph to find:

a) $f(28.50)$

b) $f(93.89)$

c) $f(8.99)$

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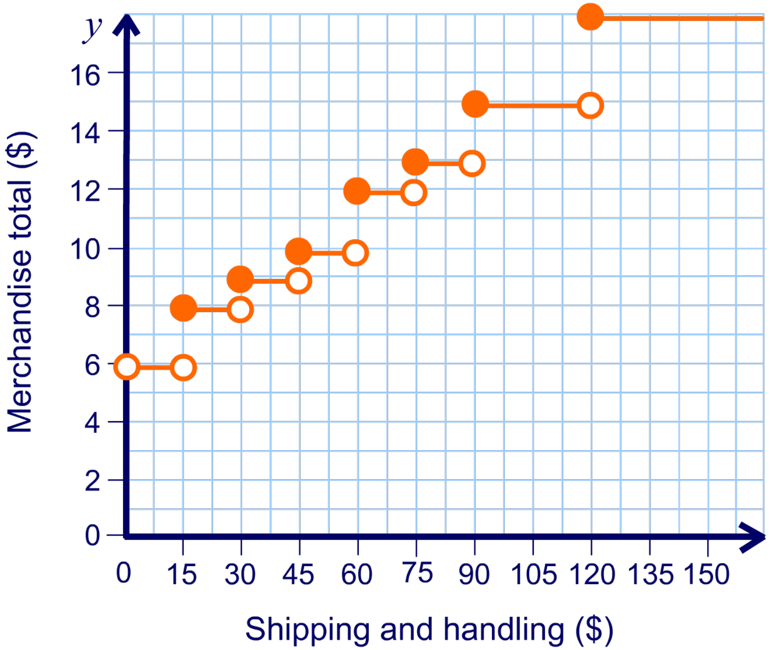
The function for the shipping cost, $f(x)$, of merchandise costing \$$x$, is:

$f(x) = 5.99$ for $0 < x < 15.00$
$f(x) = 7.99$ for $15.00 \leq x < 30.00$
$f(x) = 8.99$ for $30.00 \leq x < 45.00$
$f(x) = 9.99$ for $45.00 \leq x < 60.00$
$f(x) = 11.99$ for $60.00 \leq x < 75.00$
$f(x) = 12.99$ for $75.00 \leq x < 90.00$
$f(x) = 14.99$ for $90.00 \leq x < 120.00$
$f(x) = 17.99$ for $x \geq 120.00$.



Using your graph, the total cost including shipping for the given merchandise totals are:

a) $f(28.50) = \$7.99$, b) $f(93.89) = \$14.99$, c) $f(8.99) = £8.99$.

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The greatest integer function; $f(x) = [x]$

The **greatest integer function** (or "**floor function**") is another example of a step function. It is written as $f(x) = [x]$.

The domain is all real numbers and the range is integers.

The output value of a given input value, x, is the largest integer not greater than x. Some calculators use **int(x)**.

The graph shows $f(x) = [x]$. Use it to find:

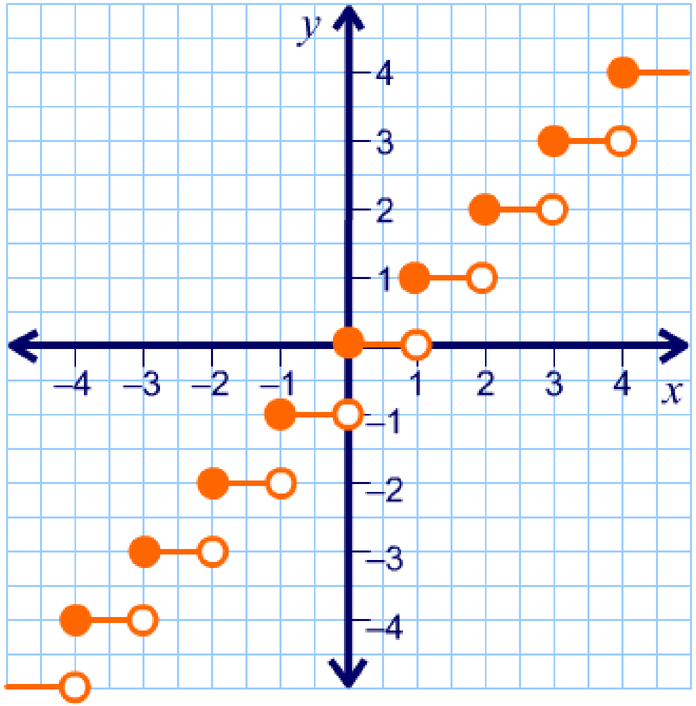
a) $f(2.3)$ 2

b) $f(0.7)$ 0

c) $f(-1.3)$ –2

d) $f(-3.7)$ –4

e) $f(3.0)$ 3



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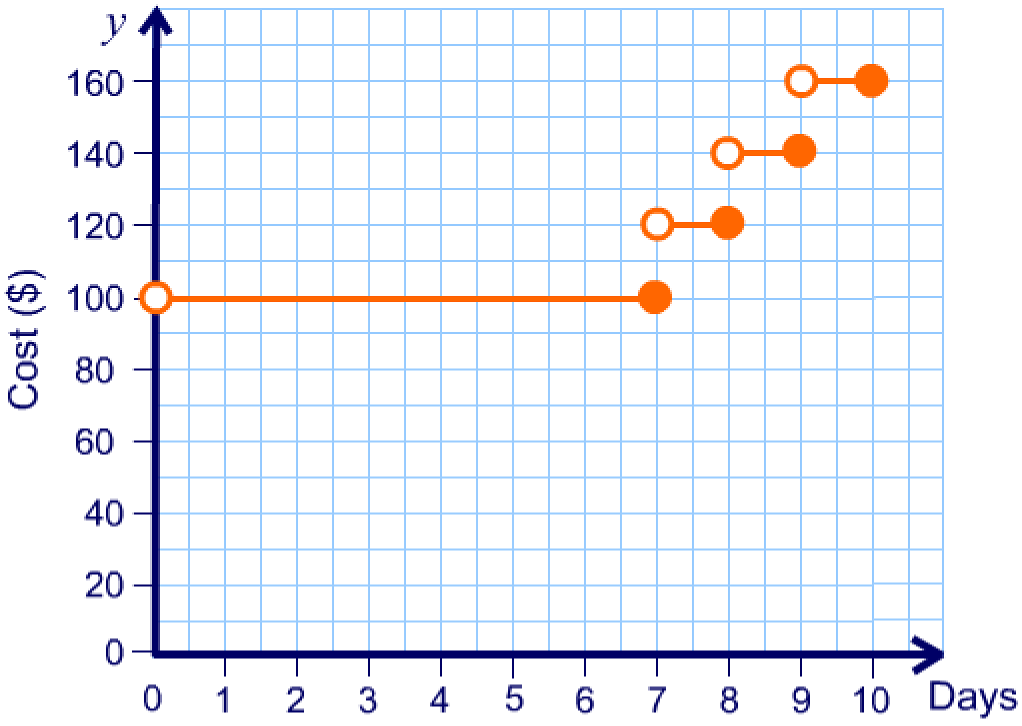
Let $C(d)$ be the function that gives the cost to park your car in the long term parking lot at the airport for d days. The sign at the entrance says: “Parking is $100.00 for up to one week, then $20.00 for each additional day, or portion thereof.”

Graph this function on graph paper.



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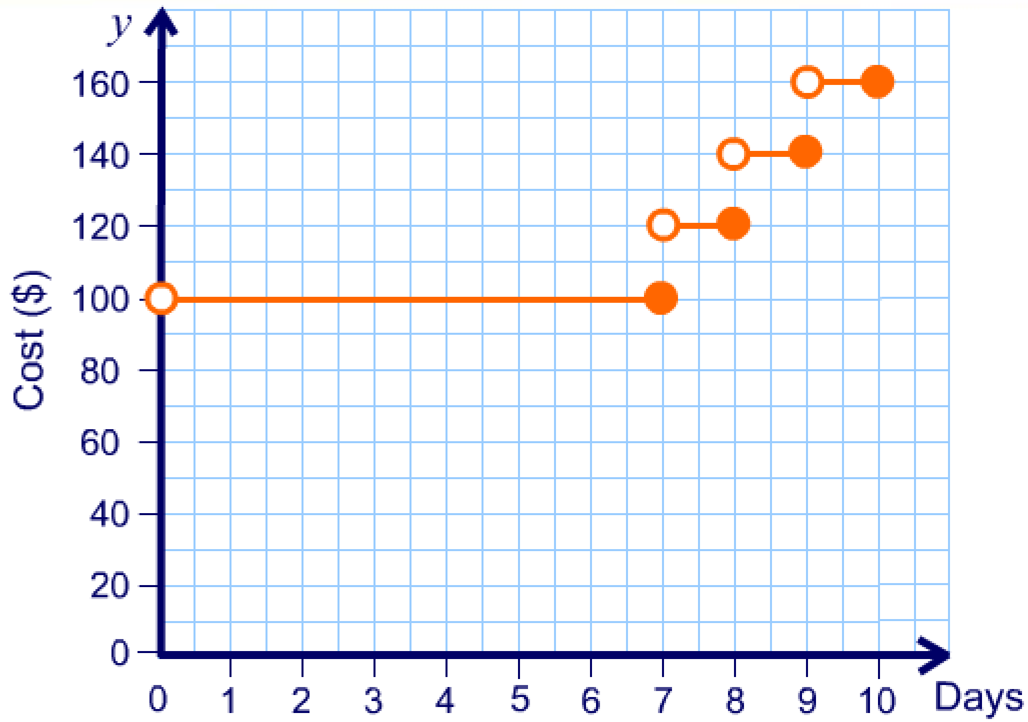
Comparing to $f(x) = [x]$



Use the graph to determine the cost for parking:

a) 9.1 days **b) 5.8 days** **c) 8.7 days**

$100 + 20(3) = **$160**

$100 (the minimum rate for 1 week or under)

$100 + 20(3) = **$160**

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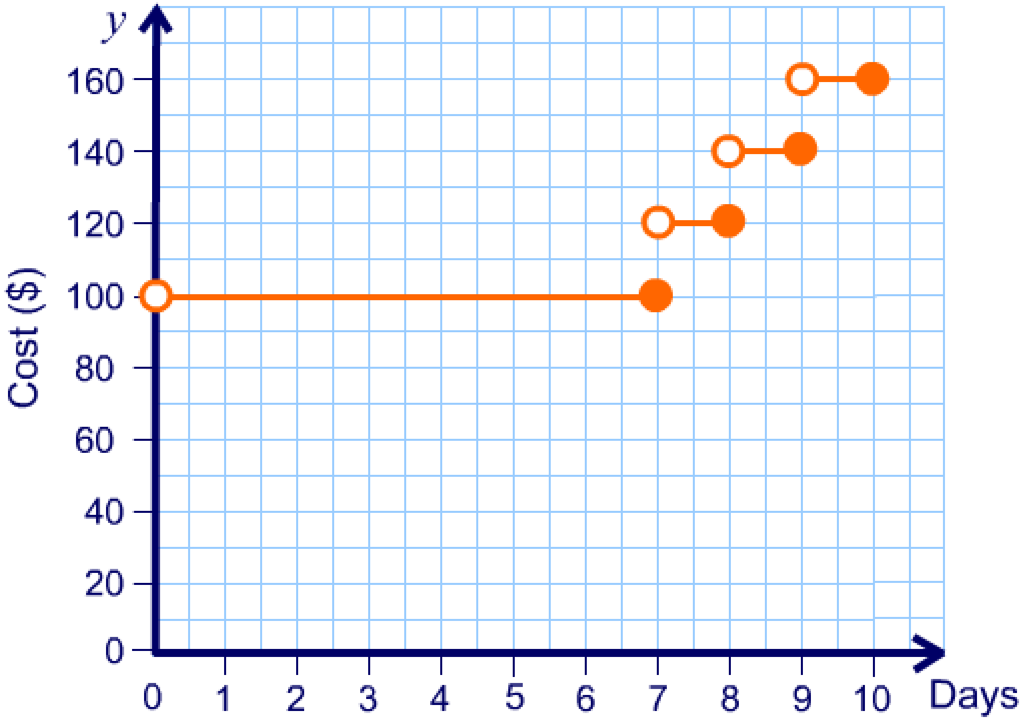
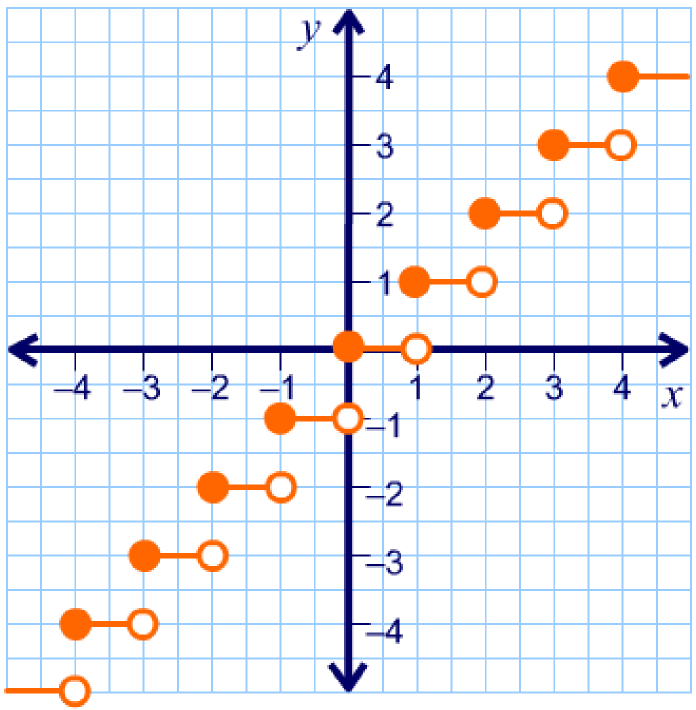
Comparing to $f(x) = [x]$

Let's compare our parking fees graph to the graph of the greatest integer function.





Describe the similarities and differences between your graph and that of the greatest integer function.

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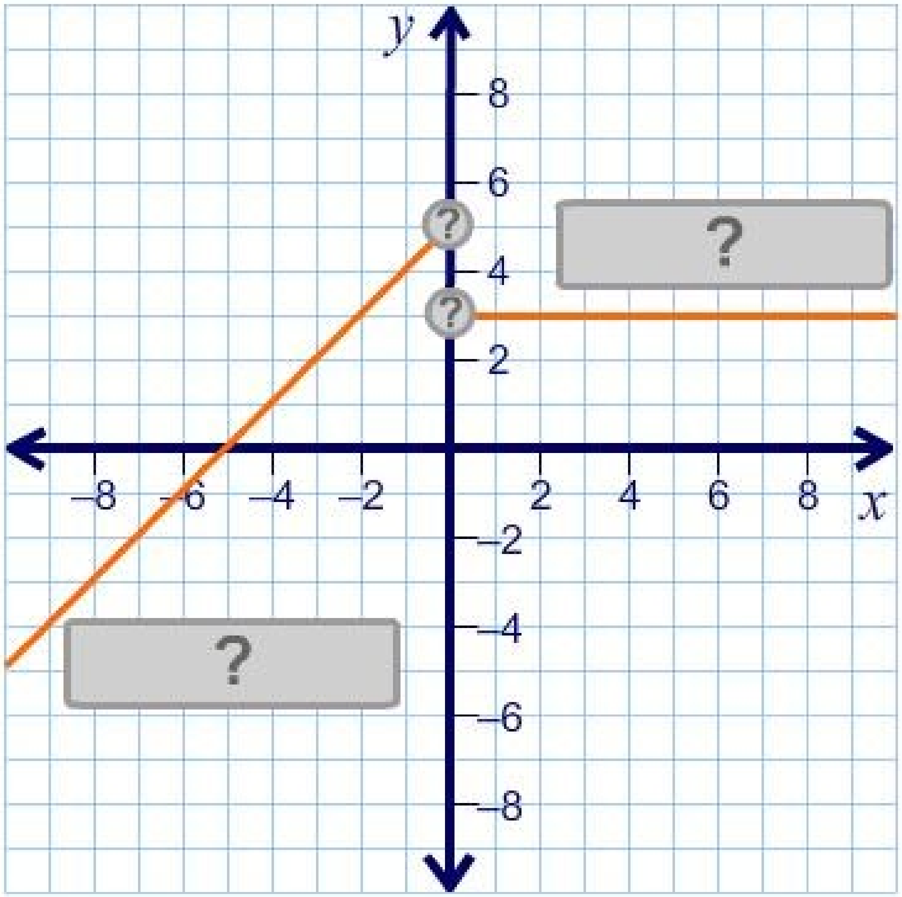

Label each part of the piecewise graph with the correct equation, and add open and closed circles to the correct points

$$f(x) = \begin{cases} x + 5, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$$

$y = x + 5$

$y = 3$

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