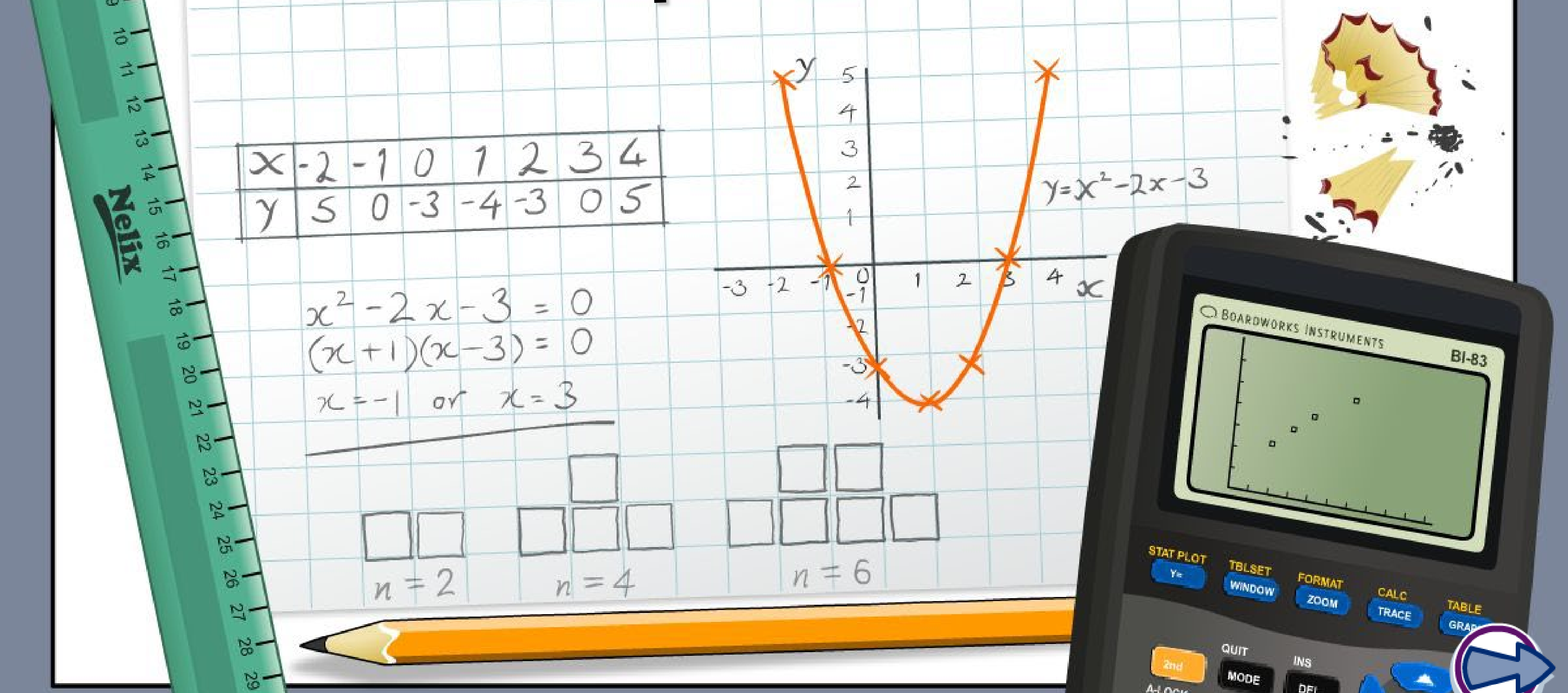
Solving linear inequalities



Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

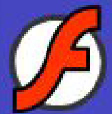
Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**



This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Solving linear inequalities

Look at the following inequality: $x + 3 \geq 7$.

This means "x **plus 3 is greater than or equal to 7**."

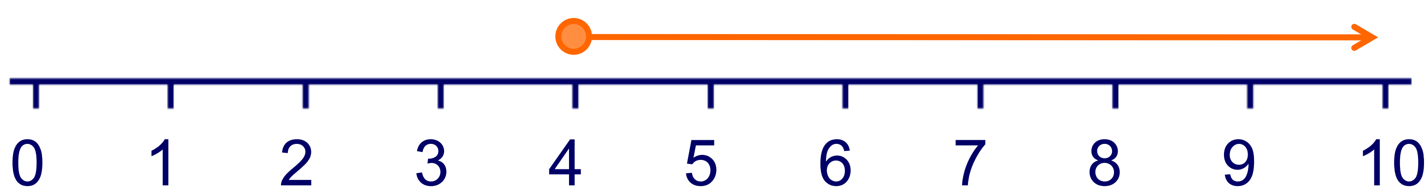
Finding the values of x that make the inequality true is called **solving the inequality**.

What values of x make this inequality true?

Adding 3 to x makes it bigger than (or equal to) 7.

x must already be bigger than 3 less than 7, otherwise adding 3 would not make it as big as 7.

Any value of x **greater or equal to 4** makes the inequality true.



Simplifying by adding or subtracting

To solve an inequality, simplify it so that it has one letter on one side of the inequality sign and a number on the other side. This is the solution to the inequality.

What values of x make $x + 3 \geq 7$ true?

Simplify the inequality by adding or subtracting the same numbers on both sides of the inequality sign.

$$x + 3 \geq 7$$

-3 $\qquad$ -3

$$x \geq 4$$

Multiplying by positive values

Like an equation, we can solve an inequality by adding or subtracting the same value to both sides of the inequality sign.

We can also multiply or divide both sides of the inequality by a **positive** value.

For example, solve $4x - 7 > 11 - 2x$

add 7 to both sides: $4x > 18 - 2x$

add $2x$ to both sides: $6x > 18$

divide both sides by 6: $x > 3$

How could we check this solution?

Checking solutions

Check that $x > 3$ is the solution to $4x – 7 > 11 – 2x$.

1. Substitute a value just above 3 into the inequality.

If we substitute $x = 4$ into the inequality we have:

$$4 \times 4 – 7 > 11 – 2 \times 4$$

$$9 > 3$$ **This is true.**

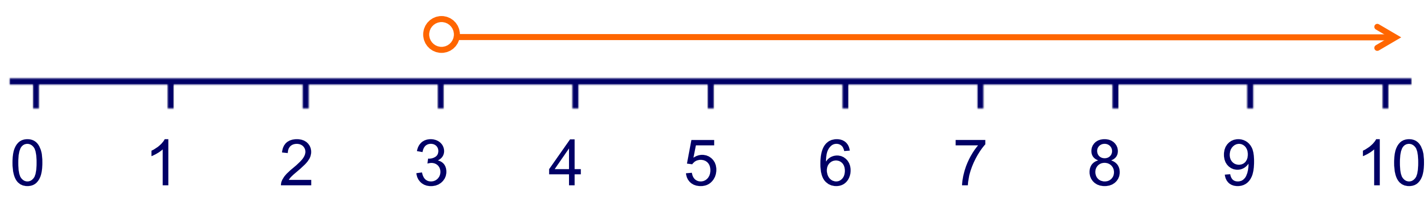
2. Substitute a value just below 3 into the inequality.

If we substitute $x = 2$ into the inequality we have:

$$4 \times 2 – 7 > 11 – 2 \times 2$$

$$1 > 7$$ **This is not true.**

Therefore the solution was correct.



Justin's homework is shown below. His teacher has marked one question. Can you mark the rest? Substitute values into the inequalities to check them.

1)	$2x + 4 < 3$	$x < -0.5$	✓
2)	$5 \geq x - 1$	$4 \geq x$	✗ $6 \geq x$
3)	$3x - 7 \geq 2$	$x \leq 6$	✗ $x \geq 3$
4)	$2x > 4$	$x > 2$	✓
5)	$-6 < 7x + 8$	$1 < 2x$	✗ $-2 < x$

Solving linear inequalities

Question 1/5: Solve the inequality and check your answer:

$$2x + 1 > x + 6$$

Press the "=" button to show the calculations step by step.

- $x > 5$
- $x < 5$
- $x \leq 5$
- $x \geq 5$

Sofia has \$100. She wants to buy two pairs of shoes at \$29 each, and as many pairs of socks as possible at \$4 each.

How many pairs of socks can Sofia buy?

How many pairs of socks could Sofia buy if she only had \$80?

How many pairs of socks can Sofia buy?

Write an inequality for the number of pairs of socks, n.

cost of shoes — cost of socks

pairs of shoes → $2 \times 29 + 4n \leq 100$ ← money available

$$n \leq 10.5$$

Sofia can buy 10 pairs of socks.

How many pairs of socks could Sofia buy if she only had $80?

$$2 \times 29 + 4n \leq 80$$

$$n \leq 5.5$$

Sofia could buy 5 pairs of socks.

Party

Joaquin has $50 for a party.

He wants to buy 3 bottles of lemonade at $1.28 each and as many bags of chips as possible at $2 each.

How many bags of chips can Joaquin afford?

The shop puts on an offer: now if you buy two bags of chips, you get a third bag free.

How many bags of chips can Joaquin afford now?

How many bags of chips can Joaquin afford?

$$3 \times 1.28 + 2n \leq 50$$

$$n \leq 23.08$$

Joaquin can afford **23 bags of chips**.

How many bags of chips can Joaquin afford using the offer of buy two, get a third free?

Joaquin can afford 23 bags of chips, which is 11 pairs. For each pair, he gets a free bag of chips, so he gets 11 bags free. This makes a total of **34 bags of chips**.

Multiplying or dividing by negatives

Although most inequalities can be solved like equations, we have to take great care when multiplying or dividing both sides of an inequality by a negative value.

The following simple inequality is true:

$$-3 < 5$$

Look what happens if we multiply both sides by –1.

$$-3 \times -1 < 5 \times -1$$

~~3 < –5~~ **This is not true.**

When we multiply or divide by a negative number we have to **reverse** the inequality sign.

$$3 > -5$$ ✓

Examples

For example, $4 - 3x \leq 10$

subtract 4 from both sides: $-3x \leq 6$

divide both side by –3: $\boldsymbol{x \geq -2}$

The inequality sign is reversed.

We could also solve this type of inequality by moving the x terms to the right so that they are positive.

$4 - 3x \leq 10$

add $3x$ to both sides: $4 \leq 10 + 3x$

subtract 10 from both sides: $-6 \leq 3x$

divide both sides by 3: $\boldsymbol{-2 \leq x}$

Solving combined linear inequalities

The two inequalities $4x + 3 \geq 5$ and $4x + 3 < 15$ can be written as a single combined inequality.

$$5 \leq 4x + 3 < 15$$

We can solve this inequality as follows:

subtract 3 from each part: $2 \leq 4x < 12$

divide each part by 4: $0.5 \leq x < 3$

Perform the same operations on all three parts of the inequality.

We can illustrate this solution on a number line as:

More combined linear inequalities

Some combined inequalities contain variables in more than one part.

For example, $x - 2 \leq 3x + 2 \leq 2x + 7$

Treat this as two separate inequalities,

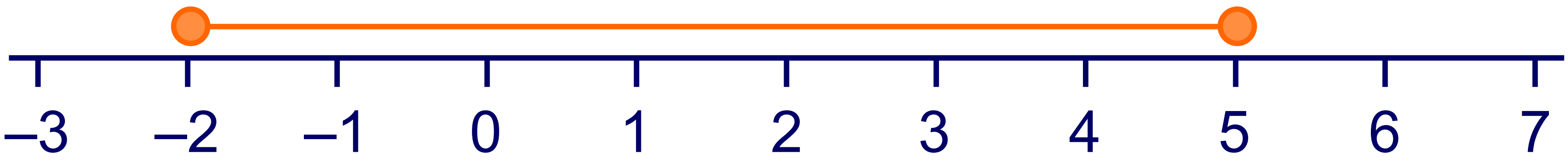
$$x - 2 \leq 3x + 2 \quad \text{and} \quad 3x + 2 \leq 2x + 7$$

$$-2 \leq 2x + 2 \qquad\qquad x + 2 \leq 7$$

$$-4 \leq 2x \qquad\qquad x \leq 5$$

$$-2 \leq x$$

We can write the complete solution as $-2 \leq x \leq 5$ and illustrate it on a number line as:



Overlapping solutions

Solve the following inequality and illustrate the solution on a number line:

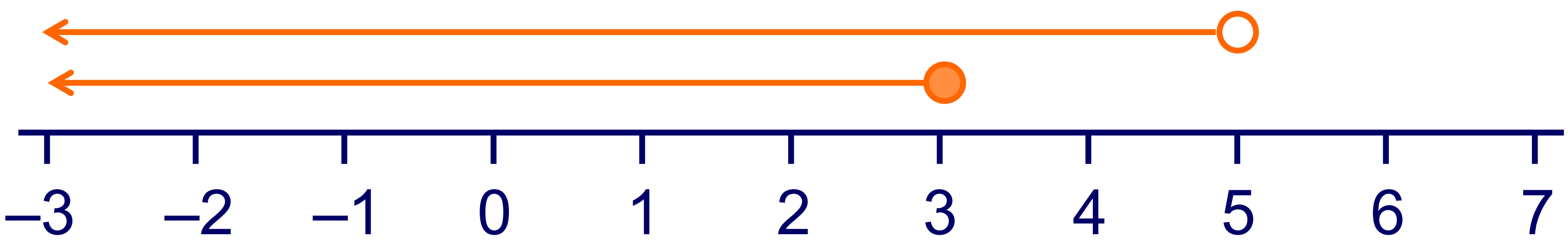
$$2x - 1 \leq x + 2 < 7$$

Treating as two separate inequalities,

$2x - 1 \leq x + 2$ and $x + 2 < 7$

$x - 1 \leq 2$ $\quad$ $x < 5$

$x \leq 3$



Only the area where both solutions overlap works for both parts of the inequality, so the solution set is given by $x \leq 3$.

Inconsistent inequalities

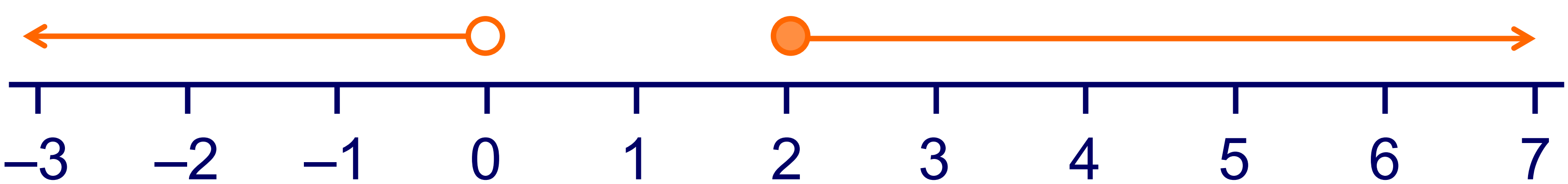
Solve the following inequality and illustrate the solution on a number line:

$$4x + 5 < 3x + 5 \leq 4x + 3$$

Treating as two separate inequalities,

$4x + 5 < 3x + 5$	and	$3x + 5 \leq 4x + 3$
$4x < 3x$		$5 \leq x + 3$
$x < 0$		$2 \leq x$
		$x \geq 2$

There are no values of x that satisfy both inequalities, so there are **no solutions**. The sets do not overlap on the number line.



Absolute value inequalities

Absolute value inequalities

The absolute value of a number is its distance from zero on a number line. When an inequality contains an absolute value, it is really two inequalities.

Select one of the examples to find out more, or "**summary**" to see the general rules for inequalities with absolute values.

$|x| < 5$ | $|x - 7| \leq 2$ | $|x| \geq 4$ | $|x + 3| > 1$ | **summary**