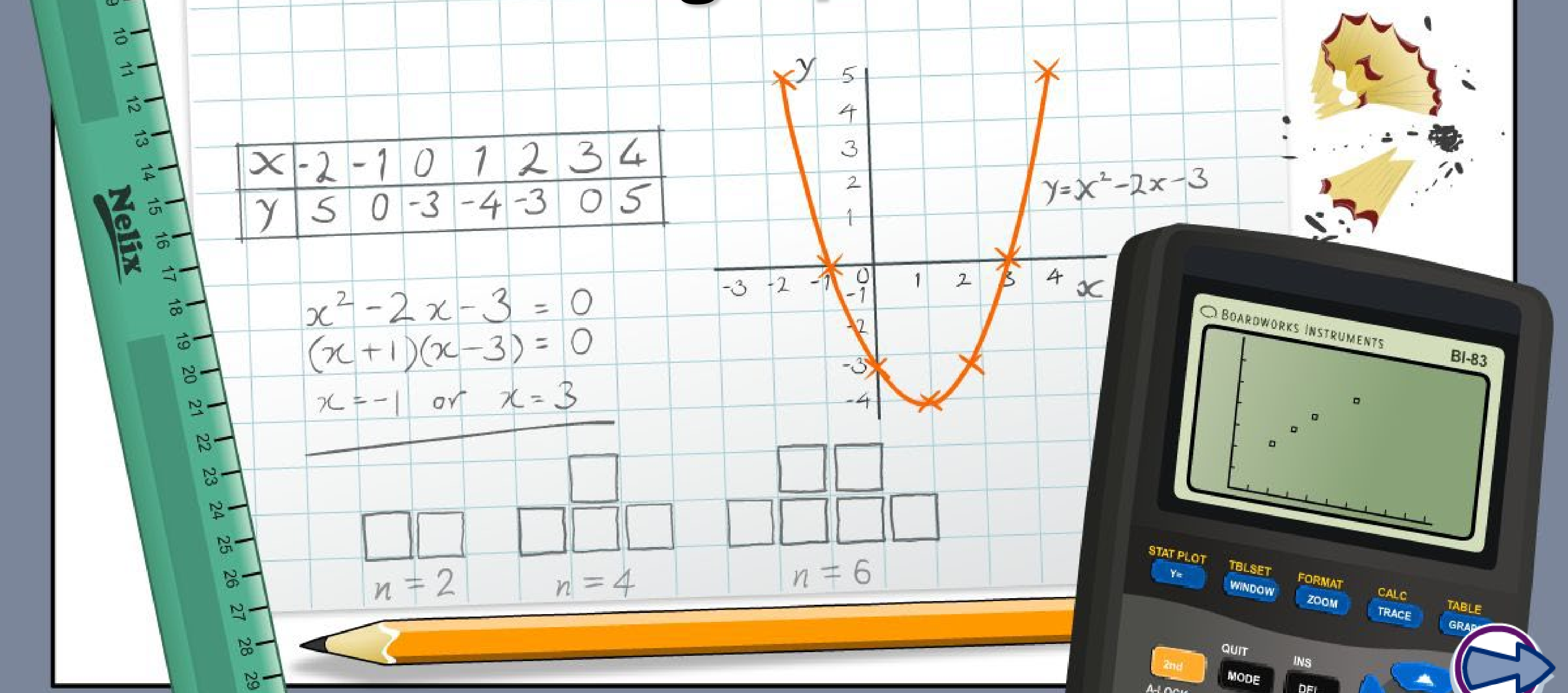
Systems of equations and graphs



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

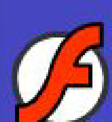
Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**



This icon indicates that the slide contains activities created in Flash. These activities are not editable.

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Equations with two variables have an infinite number of solution pairs.

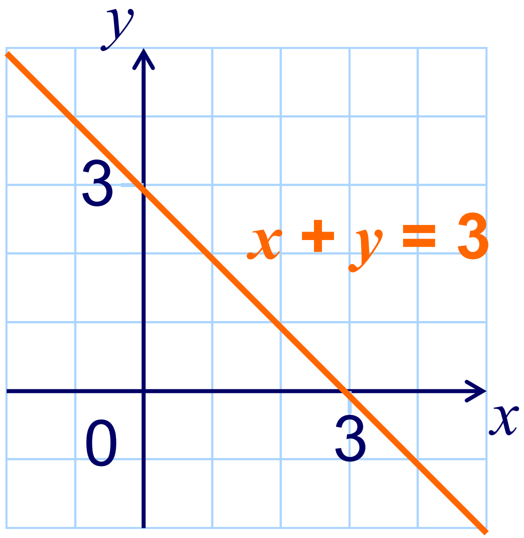
For example, $x + y = 3$ is true when

$x = 1$ and $y = 2$

$x = 3$ and $y = 0$

$x = –2$ and $y = 5$ and so on …

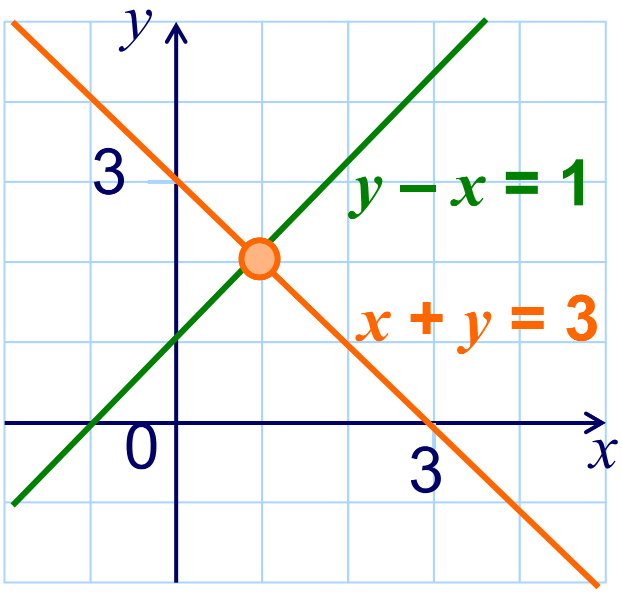
We can represent the set of solutions on a graph:



If we have two non-equivalent linear equations, both with two variables, then there is at most one pair of values that solves both these equations:

For example, $x + y = 3$ and $y - x = 1$

We can find the pair of values that solves both by drawing the lines $x + y = 3$ and $y - x = 1$ on the same graph.



The point where the two lines intersect gives the solution to both equations.

This is the point (1, 2).

The solution is $x = 1$ and $y = 2$.

Checking solutions

After using a graph to find a solution of a pair of linear equations, it is important to check the solution.

Check whether (1, 2) is a solution to both of these equations: $x + y = 3$ and $y - x = 1$.

Substitute $x = 1$ and $y = 2$ into the original equations.

$x + y = 3$

$1 + 2 = 3$

This is true.

$y - x = 1$

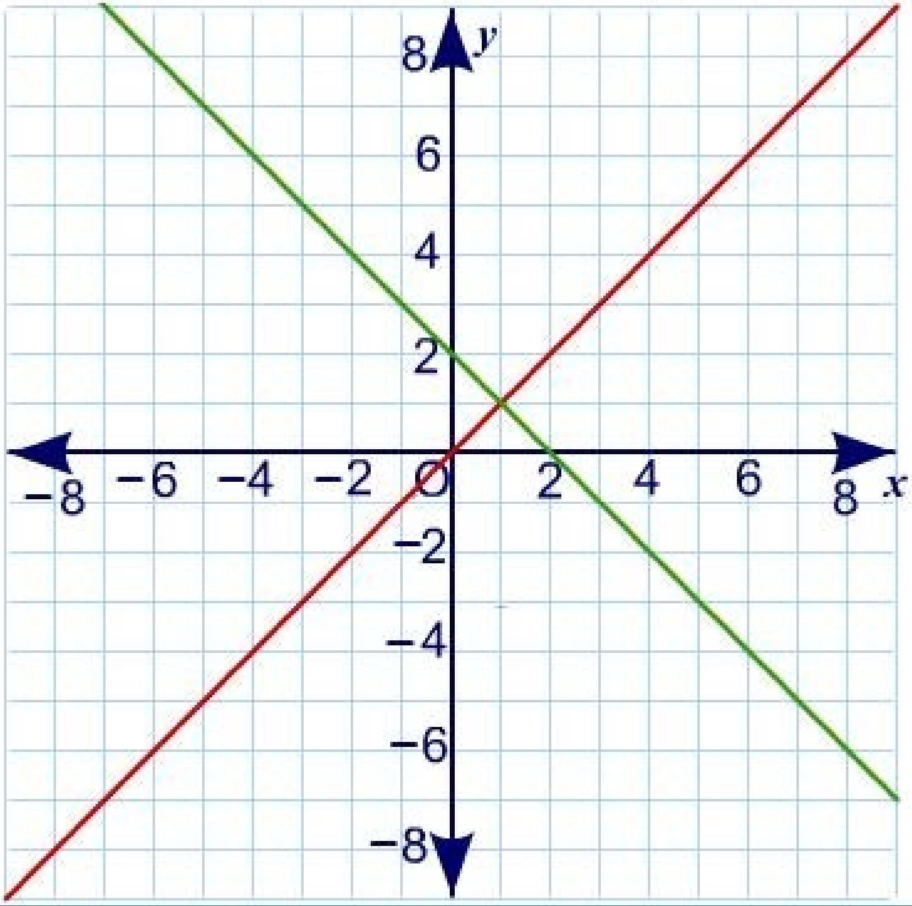
$2 - 1 = 1$

This is true.

Both of the equations are satisfied, so the solution is correct.

Check whether (0, 1) is a solution to $x - y = 2$ and $2x + y = 1$.

What is the point of intersection of the two equations?



$1x - 1y = 0$

$1x + 1y = 2$

Intersection:

Systems of equations with no solutions

Sometimes pairs of linear equations produce graphs that are parallel. Parallel lines never intersect, so there is no solution.

How can we tell whether the graphs of two lines are parallel without drawing them?

Two lines are parallel if they have the same **slope**.

We can find the slope of the line given by a linear equation by rewriting it in the form $y = mx + b$.

The value of the slope is given by the value of the constant m.

Example with no solutions

Show that the system of equations

$$y - 2x = 3$$

$$2y = 4x + 1$$

has no solutions.

Rewrite the equations in the form $y = mx + b$.

$y - 2x = 3$ | $2y = 4x + 1$

$y = 2x + 3$ | $y = 2x + \frac{1}{2}$

The slope m is **2** for both equations.

This means that the two lines are parallel, so there are no solutions.

Infinite solutions

Sometimes equations are represented by the same graph. The lines **coincide** and the equations are **equivalent**.

For example, $2x + y = 3$

$$6x + 3y = 9$$

Notice that each term in the second equation is 3 times the value of the corresponding term in the first equation.

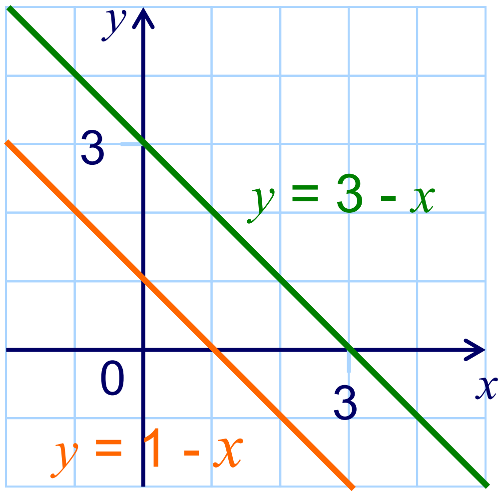
Both equations can be rearranged to give:

$$y = -2x + 3$$

When all the equations in a system can be rearranged to give the same equation, the system has an infinite number of solutions.

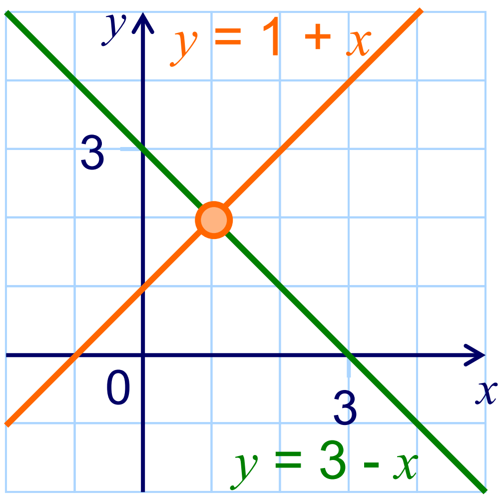
Summary

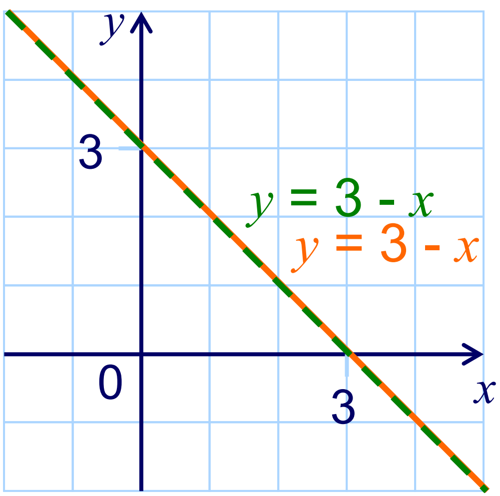
For the pair of linear equations $y = m_1x + b_1$, $y = m_2x + b_2$:



If $m_1 = m_2$ and $b_1 \neq b_2$ then the lines are parallel and there is **no solution**.

If $m_1 \neq m_2$ then the lines intersect and there is **one solution**.





If $m_1 = m_2$ and $b_1 = b_2$ then the lines coincide and there is an **infinite solution** set.

How many solutions does this pair of linear equations have?

$$2x + 7y = 13$$
$$-3x + 7y = -37$$

reminder