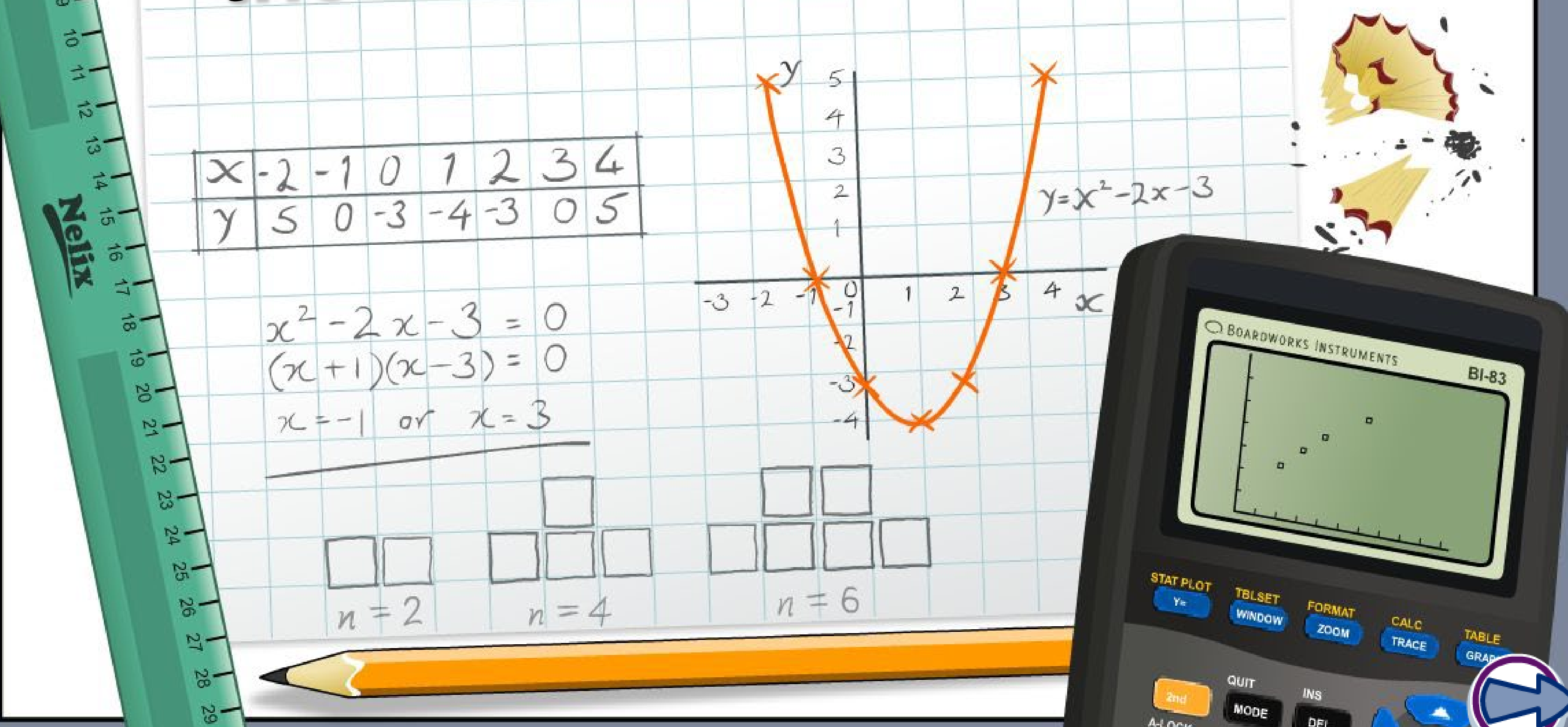
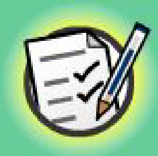
Systems of equations and the elimination method



Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

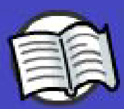
5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

Suppose that $x + y = 3$.

How many pairs of values make this equation true?

$x = 1$ and $y = 2$, $x = 3$ and $y = 0$, $x = –2$ and $y = 5$… and so on.

Equations that contain two unknowns have an **infinite** number of solution pairs.

Suppose we have **two** of these sorts of linear equations in two variables, for example, $x + y = 3$ and $y – x = 1$.

How many pairs of values satisfy *both* equations?

There is just **one** solution pair: $\mathbf{x = 1}$ and $\mathbf{y = 2}$.

Systems of equations

When we have two or more equations containing the same unknowns, we call them a **system of equations**.

For the equations $y = x - 1$ and $y = 2x - 3$, the solution is: $x = 2$ and $y = 1$.

Do the same values that satisfy these equations also satisfy the equation $2y = 3x - 4$? What do you notice about this equation?

This third equation is the **sum** of the first two equations. It is satisfied by the same pair of values.

Now add the equations $2y = x - 4$ and $y = -x + 6$. What do you notice?

We know how to solve an equation in **one** variable; we use inverse operations.

If we add two linear equations and one of the variables gets eliminated, we get **one** equation in **one** variable.

…We know how to solve one of these! We can then use the first variable to find the second variable.

Can you use this method to solve the equations $5x + 2y = 9$ and $3x - 2y = -1$?

$5x + 2y = 9$

$3x - 2y = -1$

add →

$8x = 8$

This gives $x = 1$.

Now use this to find the value of y.

The elimination method

We have the equations $5x + 2y = 9$ and $3x - 2y = -1$ and we have found that $x = 1$.

If we substitute $x = 1$ into one of the equations, we can find the corresponding value of y. Let's use $5x + 2y = 9$.

	$5x + 2y = 9$
substitute in:	$5(1) + 2y = 9$
simplify:	$5 + 2y = 9$
subtract 5:	$2y = 4$
divide by 2:	$\mathbf{y = 2}$

How can we check the solution?

We can check the solution by substituting $x = 1$ and $y = 2$ into one of the equations (either $5x + 2y = 9$ or $3x - 2y = -1$.)

Which equation do you think we should use to check the solution? Why?

We used the equation $5x + 2y = 9$ to find the value of y. The solution must satisfy *both* equations, so it is important to check our overall solution using $3x - 2y = -1$ instead.

$$3(1) - 2(2) = -1$$

$$3 - 4 = -1 \checkmark$$

This is true, so we know that our solutions are correct.

This method of solving systems of equations is called the **elimination** method.

Subtracting the equations

How could we use the elimination method to solve the following two equations?

$3y = 2x + 9$ (A) and $y = 2x - 5$ (B)

Both equations contain $2x$, but both are positive, so adding them at this stage would not eliminate x.

Instead, we can **subtract** one equation from the other.

Subtracting (B) from (A):

$$\begin{array}{r} 3y = 2x + 9 \\ -\quad y = 2x - 5 \\ \hline 2y = 14 \end{array}$$

divide by 2: $\mathbf{y = 7}$

It does not matter which you subtract from which, but choose the way that will give the easiest numbers to work with.

Subtracting the equations

Now we substitute $y = 7$ into either of the equations
$3y = 2x + 9$ (A) or $y = 2x - 5$ (B) to find the value of x.

Using $y = 2x - 5$ we get:

substitute in:	$7 = 2x - 5$
simplify:	$7 + 5 = 2x$
divide by 2:	$12 = 2x$
divide by 2:	$\mathbf{6 = x}$

Check the solution using $3y = 2x + 9$.

$3(7) = 2(6) + 9$

$21 = 12 + 9$ ✓

Solve these systems of equations by adding or subtracting them.

$$10x + 6y = -75$$

$$-10x + 7y = -55$$

$x =$

$y =$

Practice questions: the elimination method

1.	Solve $x + 2y = 16$ and $x - 8y = 6$.
2.	Solve $-x - 4y = 11$ and $x + y = 4$.
3.	Solve $3x + 7y = 22$ and $3x + 4y = 10$.
4.	Solve $2x + 5y = 8$ and $3x + 5y = 17$.
5.	Solve $2x + 6y = 4$ and $2x - 6y = 16$.

The elimination method

How could we use the elimination method to solve the following two equations?

$4x - y = 29$ Ⓐ and $3x + 2y = 19$ Ⓑ

Sometimes we need to **multiply** one of the equations by a constant before we can eliminate one of the variables.

We need it so that there is the same number in front of either the x or the y before adding or subtracting the equations.

Here, we can multiply equation Ⓐ by 2:

2 × Ⓐ : $8x - 2y = 58$ Ⓒ

Ⓑ : $3x + 2y = 19$

Call this equation Ⓒ

We now have two equations that can be added to eliminate y.

The elimination method

$$\begin{array}{rrl} & 8x - 2y = 58 & \text{(C)} \\ + & 3x + 2y = 19 & \text{(B)} \\ \hline & 11x \quad\quad = 77 & \end{array}$$

divide by 11: $\mathbf{x = 7}$

To find the value of y when $x = 7$, substitute this value into one of the equations:

Substituting $x = 7$ into (B) gives:

$3(7) + 2y = 19$

simplify: $21 + 2y = 19$

subtract 21: $2y = -2$

divide by 2: $\mathbf{y = -1}$

Remember to check your solution by substituting into (C).

How could we use the elimination method to solve the following two equations?

$-4x - 2y = 10$ (A) and $7x + 3y = -6$ (B)

Sometimes we need to multiply **both** of the equations by a different constant before we can eliminate a variable.

As always, we need it so that there is the same number in front of either the x or the y.

Here, we can multiply equation (A) by 3 and equation (B) by 2:

$3 \times$ (A) : $-12x - 6y = 30$ (C) — Call this equation (C)

$2 \times$ (B) : $14x + 6y = -12$ (D) — Call this equation (D)

We now have two equations that can be added to eliminate y.

The elimination method

$$
\begin{array}{rl}
-12x - 6y = 30 & \text{(C)} \\
+ \quad 14x + 6y = -12 & \text{(D)} \\
\hline
2x \qquad = 18 &
\end{array}
$$

divide by 2: $\mathbf{x = 9}$

To find the value of y when $x = 9$, substitute this value into one of the equations:

Substituting $x = 7$ into (D) gives:

$$14(9) + 6y = -12$$

simplify: $126 + 6y = -12$

subtract 19: $6y = -138$

divide by 6: $\mathbf{y = -23}$

Remember to check your solution by substituting into (C).

More elimination practice

Solve these systems of equations by adding or subtracting them.

$10x + 8y = -17$ → × 1 → $10x + 8y = -17$

$2x + 2y = -2$ → × 1 → $2x + 2y = -2$

$x =$

$y =$

Practice questions: the elimination method

1.	Solve $3x - 2y = 4$ and $2x + y = 12$.	?	W
2.	Solve $-2x + 3y = 10$ and $x - y = 6$.	?	W
3.	Solve $x - 2y = 9$ and $5x - 3y = 10$.	?	W
4.	Solve $-2x + 5y = 10$ and $x + 3y = 6$.	?	W
5.	Solve $2x - 2y = 24$ and $5x + 3y = 20$.	?	W

Special cases

Case 1 | Case 2

BE CAREFUL!

Not all systems of equations have one solution.

Press on each of the tabs above to see an example of other cases to look out for.

Basketball quiz

Practice identifying solutions to systems of equations in this team quiz! Get into two teams: A and B.

Each team will be represented by a basketball player. If your team answers a question correctly, your player scores a point. The team with the highest score wins!

Press **start** to begin.

start