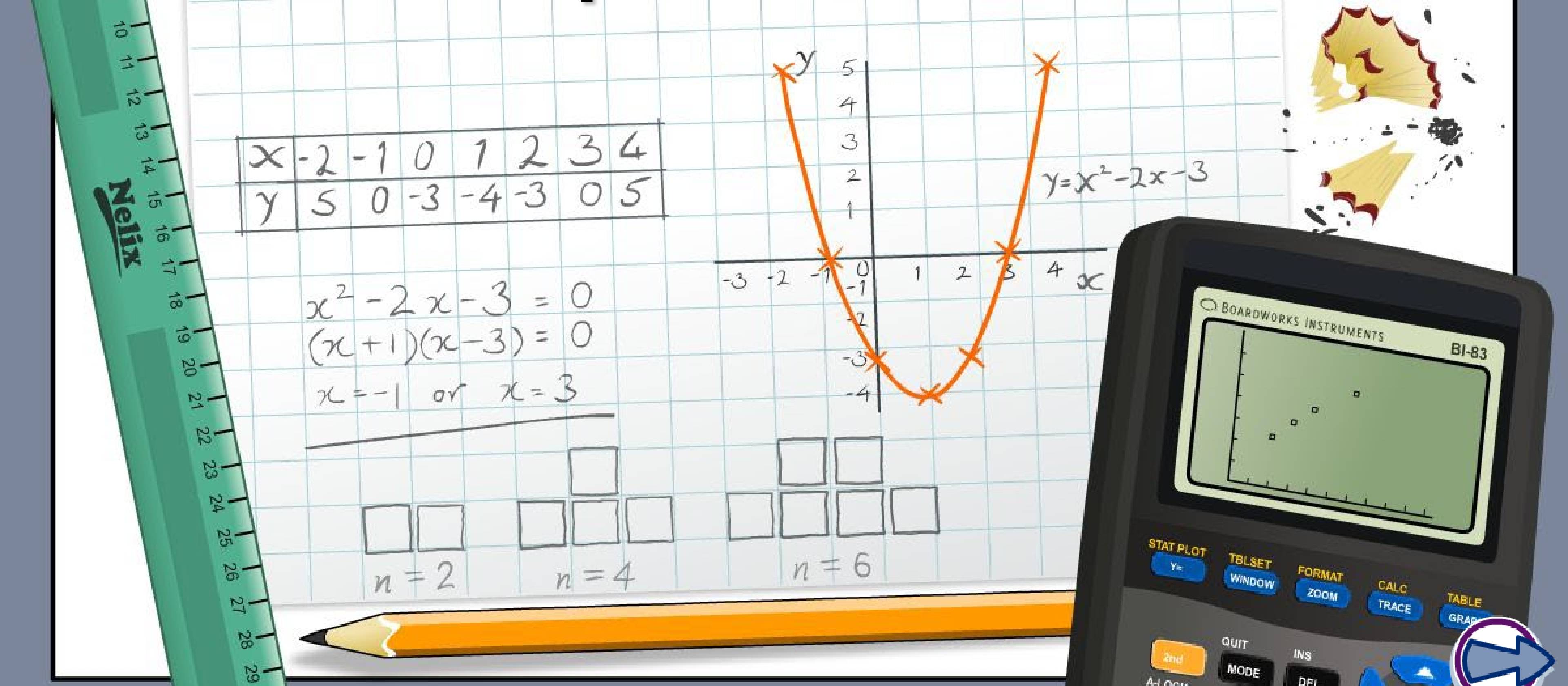
The range and interquartile range



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

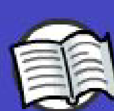
5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.



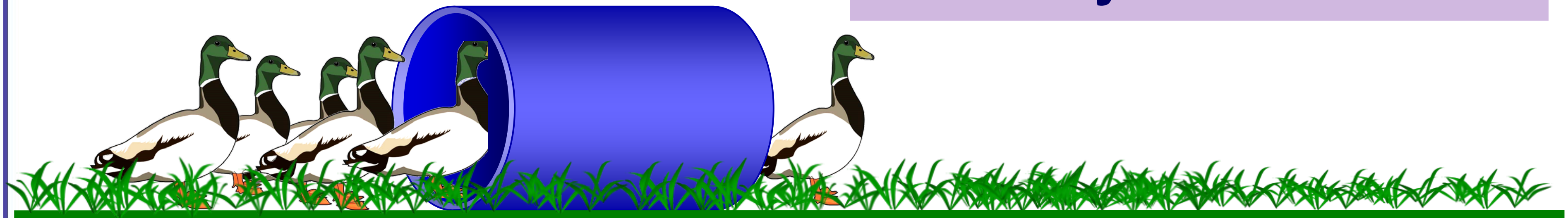
This icon indicates teacher's notes in the Notes field.

Find the range

Here are Laura and Jo’s duck herding times in minutes:

Laura	6.59	6.45	9.41	5.10	7.30
Jo	8.52	7.41	8.35	6.20	5.15

In groups, discuss which girl is more consistent. How did you decide this?



- If the scores are **spread out** then the range will be higher and the scores are **less consistent**.
- If the scores are **close together** then the range will be lower and the scores are **more consistent**.

The spread is called the **range**.
range = highest score – lowest score

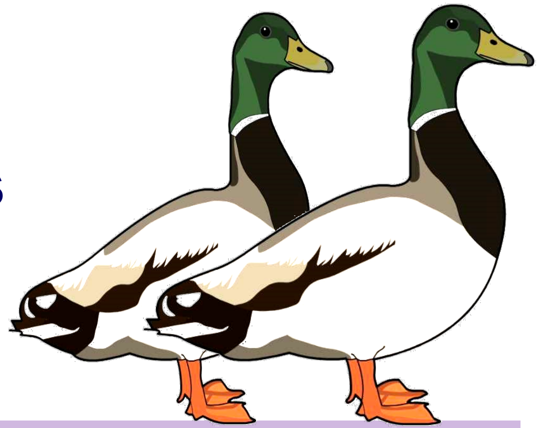
Find the range

Calculate the range for each girl to confirm the result of your discussion.

Laura	6.59	6.45	9.41	5.10	7.30
Jo	8.52	7.41	8.35	6.20	5.15

Laura's range = 9.41 – 5.10 = **4.31 mins**
Jo's range = 8.52 – 5.15 = **3.37 mins**

Jo is more consistent.



Now calculate the median for each girl.
Which girl would you enter into a competition? Why?

	Laura	Jo
range	4.31 mins	3.37 mins
median	6.59 mins	7.41 mins

Remember, the range is **not** a measure of center.

Recap: mean, median and range

80	134	–31	267	108	–18	295	235	236

Mean:

Median:

Range:

next

Missing values

23	19	24	26	17	?

Find a possible value for the missing number if:

Problem 1: The median value is 22

Missing number =

next

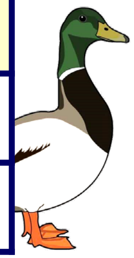
Here is a summary of Chris's and Rob's performances in 10 duck herding competitions.

	Chris	Rob
median	5.10 mins	4.26 mins
range	2.93 mins	2.18 mins

Discuss which of these conclusions are correct:

- **Rob is more reliable.**
- **Chris is better because his median is higher.**
- **Chris is better because his range is higher.**
- **Rob must have gotten a better quickest time.**
- **On average, Rob is faster and more consistent.**

	Chris	Rob
median	5.10 mins	4.26 mins
range	2.93 mins	2.18 mins



Here is the original data for Chris and Rob.

Set A:	4.26	6.29	5.14	5.17	6.11	3.36	5.54	5.06	4.29	4.31
Set B:	3.52	4.14	4.03	5.09	3.37	4.46	5.55	4.41	3.51	5.42

Use the summary table above to decide which data set is Chris's and which is Rob's.

- **Who has the best time?**
- **Who has the worst time?**

Interquartile range

When there are extreme values in the data, it is more appropriate to calculate the **interquartile range** (**IQR**).

This is the range of the **middle half** of the data, when the values are written in order.

interquartile range =
upper quartile – lower quartile

The **lower quartile** is the value that is one quarter of the way along the list.

The **upper quartile** is the value that is three quarters of the way along the list.

Extreme values in a data set are called **outliers**.

What is an outlier?

An **outlier** is defined as any data point that is more than **1.5 times the interquartile range** (IQR) below the first quartile or above the third quartile.

For the set of data values 18, 24, 26, 32, 45, 52, 96 we find the median, Q_1, Q_3, and the interquartile range in order to see if there are any outliers in the data set.

18, **24**, 26, **32**, 45, **52**, 96

Q_1 (24) **median** (32) **Q_3** (52)

$$IQR = 52 - 24 = 28$$

$$1.5 \times IQR = 1.5 \times 28 = \mathbf{42}$$

$Q_1 - 42 = 24 - 42 = \mathbf{-18}$ and $Q_3 + 42 = 52 + 42 = \mathbf{94}$.

Are there any outliers in this data?
What are they?

Girls' basketball statistics

The number of free throw points made by each team member in the most recent season for a girls' college basketball team is listed in the table on the left below.

player	free throw points
Michala	3
Lauren	4
Heather	15
Caroline	19
Brianna	19
Kiah	32
Stefanie	51
Kelly	63
Bria	65
Kaleena	70
Tiffany	142

Determine the IQR then decide if there are any outliers in the data.

$Q_1 = 15$ and $Q_3 = 65$.

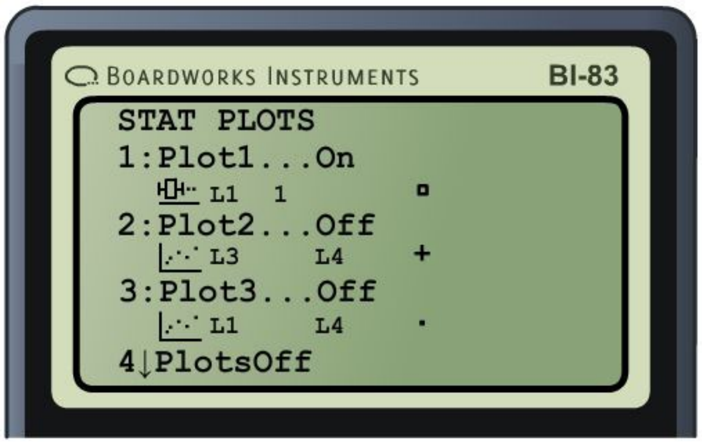
The IQR = 50 and $1.5 \times 50 = 75$

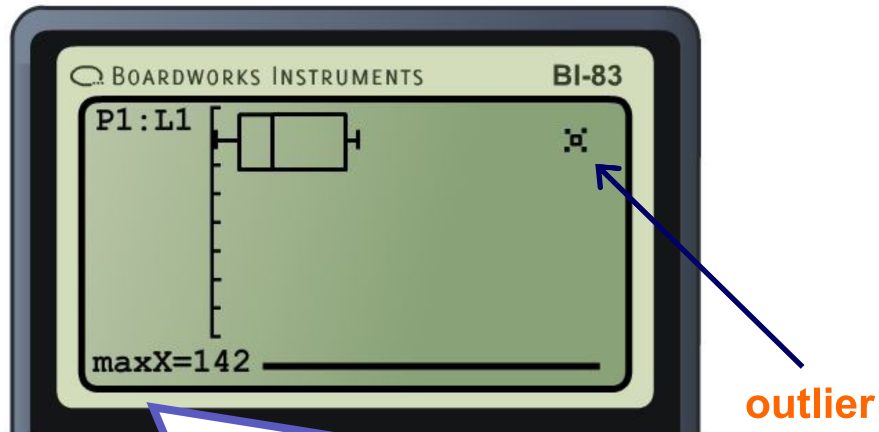
So, outliers are values less than $(Q_1 – 5)$ or greater than $(Q_3 + 75)$.

i.e. $– 60 > \text{outlier} > 140$

The data point **142** is an outlier.

Using the **STAT PLOT** feature of your graphing calculator and choosing the modified box and whisker graph produces the plot below. It indicates that the number 142 is an outlier.





Use the "trace" feature to see the value of the outlier, which is displayed in the bottom corner.

Notice that the plot 'fences in' the expected data with the outlier lying outside of the 'fence'.

Interquartile range

If there are lots of values in a data set, it is not always easy to find the interquartile range by inspection.

In these cases, there are formulas we can use.

When there are n values in an ordered data set:

$$\text{lower quartile} = \frac{n+1}{4}\text{th value}$$

$$\text{median} = \frac{n+1}{2}\text{th value}$$

$$\text{upper quartile} = \frac{3(n+1)}{4}\text{th value}$$

Interquartile range

–9, –9, –8, –6, –3, 0, 0, 1, 1, 4, 5, 5, 7, 7, 7, 9, 9, 9, 10, 10, 11, 11, 12, 12, 12, 13, 13, 14, 16, 17, 17, 18, 19, 19, 19, 20, 21, 21, 22, 22, 23, 23, 23, 23, 24, 24, 24, 25, 25, 25, 25, 26, 26, 26, 26, 27, 28, 28, 28, 29, 29, 29, 29, 30, 30, 30, 30, 30, 31, 31, 31, 33, 33, 33, 34, 34, 34, 34, 35, 35, 37, 37, 37, 37, 38, 39, 40, 40, 40, 41

Lower quartile:

Upper quartile:

Interquartile range:

next

2010	2011
57.9	61.5
62.1	62.2
63.9	64.9
75.5	74.2
81.0	87.3
98.2	95.8
114.4	115.8
119.3	125.5
136.7	139.1
140.2	148.5
220.8	149.2

The table shows the results (in seconds) of a 250 m Wife Carrying race held in Finland in 2010 and 2011.

Compare the 2010 and 2011 results using the range and interquartile range. Which gives a better summary of the data? Why?

	2010	2011
mean	106.4	102.2
range	162.9	87.7
IQ Range	72.8	74.2

Interquartile range for 2010: 136.7 – 63.9 = **72.8 s**
Interquartile range for 2011: 139.1 – 64.9 = **74.2 s**