

Arithmetic Sequences and Series

$$f(x) = x^3 - 3x^2 + x - 3$$

$$f(1) = 1^3 - 3(1)^2 + 1 - 3 = -4$$

$$f(3) = 3^3 - 3(3)^2 + 3 - 3 = 0$$

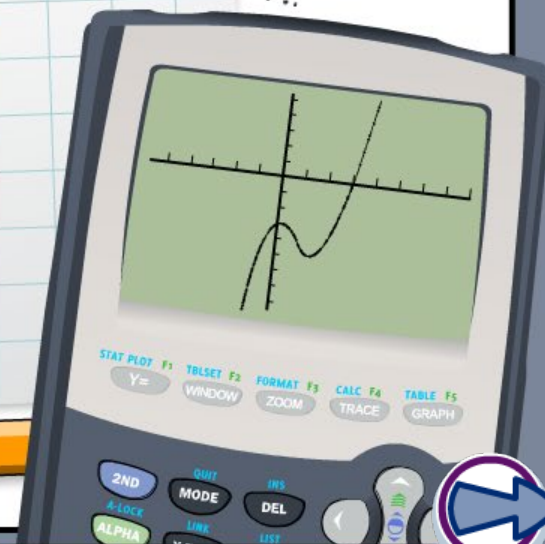
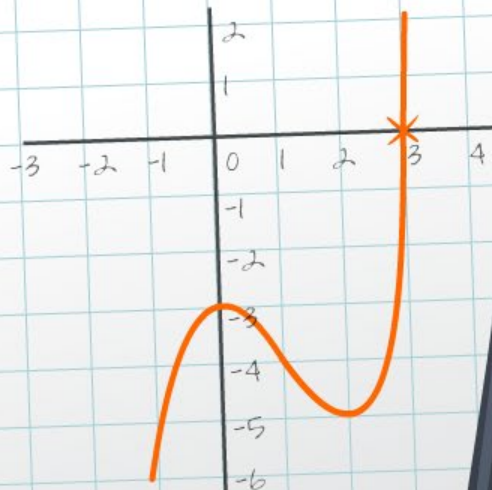
$$\begin{array}{r|rrrr} +3 & 1 & -3 & 1 & -3 \\ & & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$f(x) = x^3 - 3x^2 + x - 3$$

$$= (x - 3)(x^2 + 1)$$

$$= (x - 3)(x + i)(x - i)$$

$$x = 3 \text{ or } x = i \text{ or } x = -i$$



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.



This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

- 1) **Make sense of problems and persevere in solving them.**
- 2) **Reason abstractly and quantitatively.**
- 3) **Construct viable arguments and critique the reasoning of others.**
- 4) **Model with mathematics.**
- 5) **Use appropriate tools strategically.**
- 6) **Attend to precision.**
- 7) **Look for and make use of structure.**
- 8) **Look for and express regularity in repeated reasoning.**



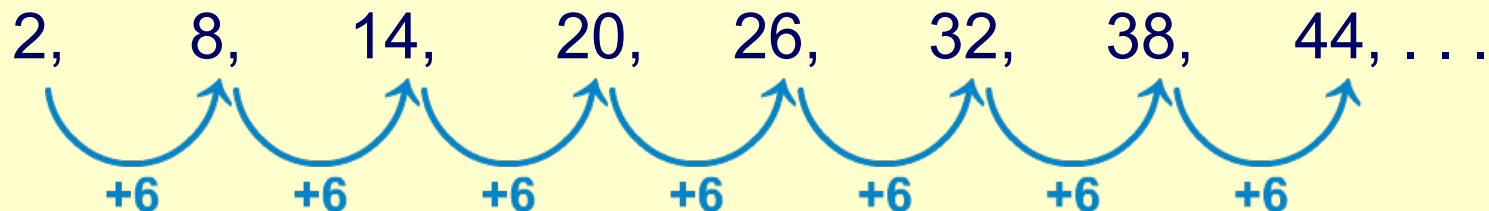
This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.



Describe the pattern of the following numbers:



This sequence starts with 2 and progresses by adding 6 to each term.

A sequence that progresses by adding a **common difference**, d , to each consecutive term is called an **arithmetic sequence**.

The formula for the n^{th} term of an arithmetic sequence with first term a_1 and common difference d is:

n^{th} term of an arithmetic sequence:

$$a_n = a_1 + (n - 1)d$$

Find the 9th term of a sequence where a_1 is 3 and d is 7.

state the formula: $a_n = a_1 + (n - 1)d$

substitute in known values: $a_9 = 3 + (9 - 1)7$
 $= 59$

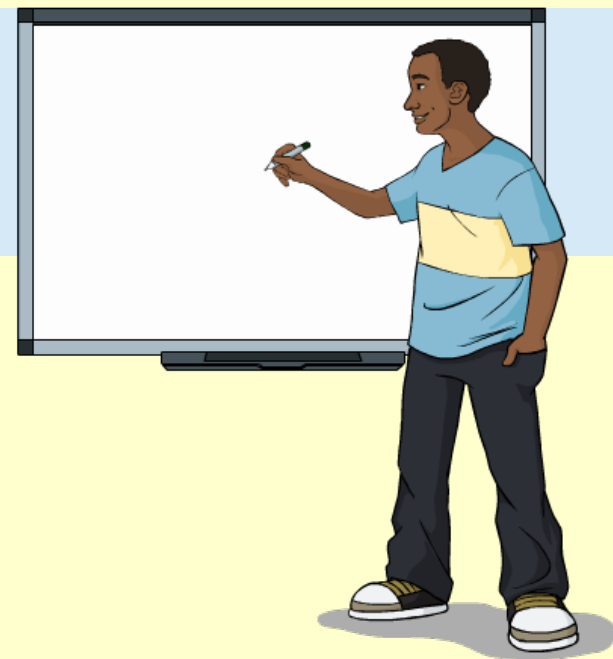
Write the formula for the n^{th} term of the following arithmetic sequence:
-9, 2, 13, 24, 35, 46 ...

find d and a_1 : $d = 2 - (-9)$
 $d = 11$

$$a_1 = -9$$

substitute into the formula: $a_n = -9 + (n - 1)11$

simplify: $= 11n - 20$





Tile patterns



pattern 1
tiles: 1



pattern 2
tiles: 5



pattern 3
tiles: 9



pattern 4
tiles: 13

How many tiles will be in the 5th pattern?

Write an equation to find the n^{th} pattern in the sequence.





The sum of a finite number of terms in an arithmetic sequence is called an **arithmetic series**. How can we write a formula for the sum of the first n terms of an arithmetic sequence?

Press **play** to see how.



The formula for an arithmetic series with n terms is:

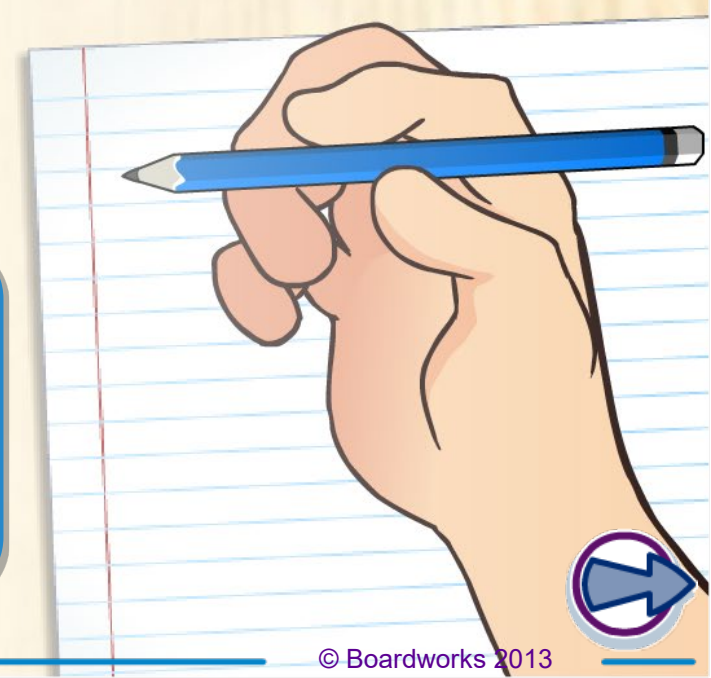
sum of an arithmetic series with n terms:

$$S_n = \frac{n(a_1 + a_n)}{2} \quad \text{or} \quad S_n = \frac{n}{2}(2a_1 + (n - 1)d)$$

The sum of an arithmetic series can also be written in summation notation:

summation notation for an arithmetic series:

$$S_n = \sum_{k=1}^n a_k = \sum_{k=1}^n a_1 + d(k - 1)$$



What is the summation notation for the arithmetic series:
 $-3 + 4 + 11 + 18 + 25 + 32 + \dots + 515 + 522$

find a_1 : -3

find d : $4 - (-3) = 7$

use the formula for an arithmetic sequence:

$$a_k = a_1 + (k - 1)d$$

substitute in a_1 and d :

$$= -3 + (k - 1)7$$

simplify:

$$= 7k - 10$$

substitute $a_n = 522$ and $k = n$ in to find the upper limit:

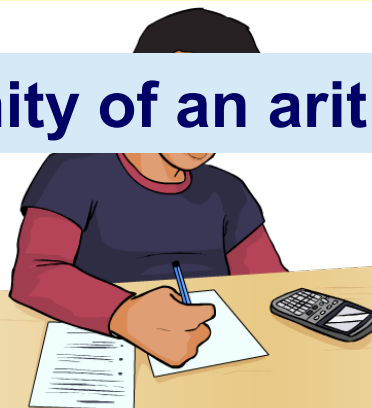
$$522 = 7n - 10$$

$$n = 76$$

complete the summation notation:

$$\sum_{k=1}^{76} 7k - 10$$

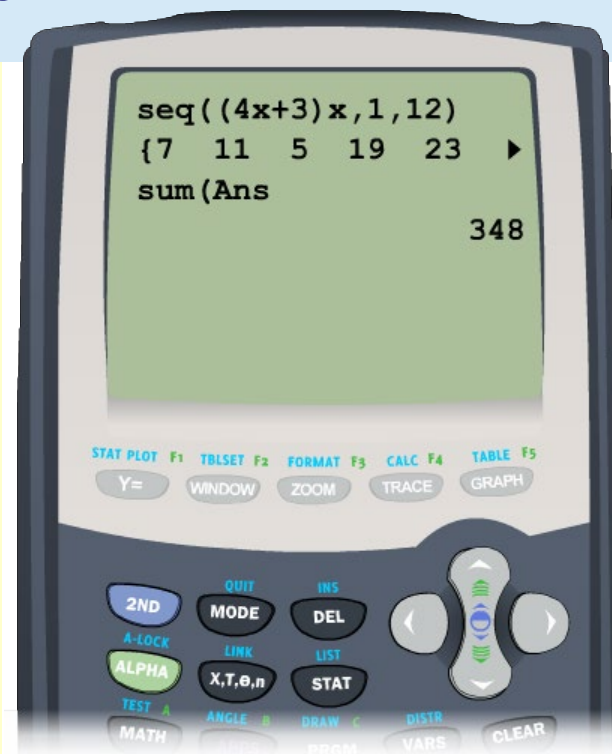
What is the sum to infinity of an arithmetic series?



A graphing calculator can be used to find the sum of a series.

Find $\sum_{k=1}^{12} 4k + 3$ using a graphing calculator.

- Press “2nd” than “STAT” to get the “LIST” menu. Scroll to the “OPS” menu and select “seq(”.
- Enter the sequence: $4x + 3$. The variable is “ x ”, start is 1 and end is 12 since $n = 12$. Press “Paste”.
- Press “ENTER” to see a list of the terms.
- To sum the terms, press “LIST” then select “sum(” on the “MATH” menu. Press “ANS” to insert your series then “ENTER” to see the sum.
- The sum is **348**.



Practice problems

1) The sum of an arithmetic series with 8 terms is -344 and the first term is -22 . What is d ?

2) Write the summation notation for the series:
 $1.25 + 2.50 + 3.75 + 5.00 + \dots + 40.00$

3) Find $\sum_{k=1}^{29} [0.3k + 0.4]$.

4) $a_1 = 18$, $a_n = 0$ and $d = -2$

Find the sum of the n terms of the series.



Jake earns \$30,000 a year in his first year at a company. Every year his salary will increase by \$2,500. What will his salary be in the 6th year?

write the formula to find a_n : $a_n = a_1 + (n - 1)d$

substitute: $a_6 = 30000 + (6 - 1)2500$
 $= \$42,500$ in year 6

How much money total will he have earned in 6 years?

write the formula to find S_n : $S_n = \frac{n(a_1 + a_n)}{2}$

substitute: $S_6 = \frac{6(30000 + 42500)}{2}$

evaluate: $= \$217,500$ total in 6 years

