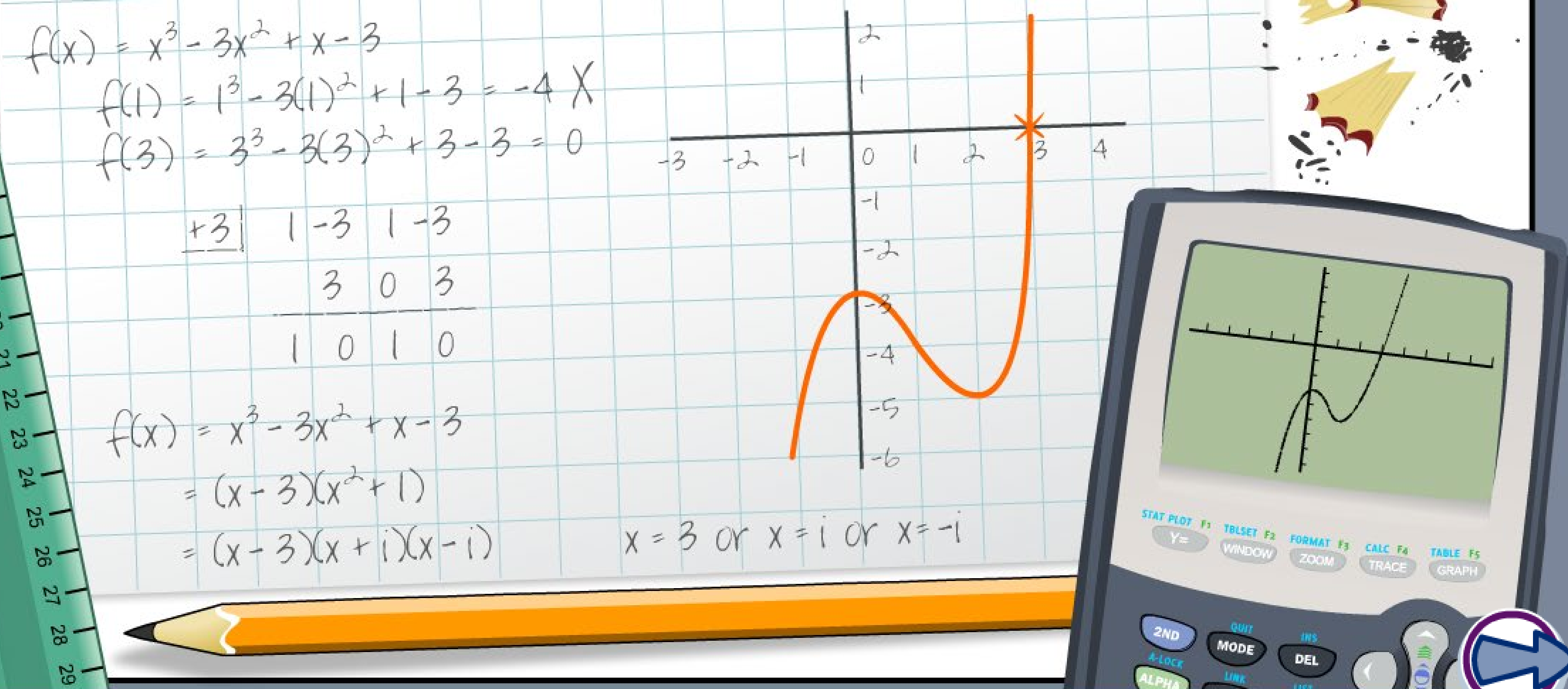
Inverse Functions



Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

MODELING

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

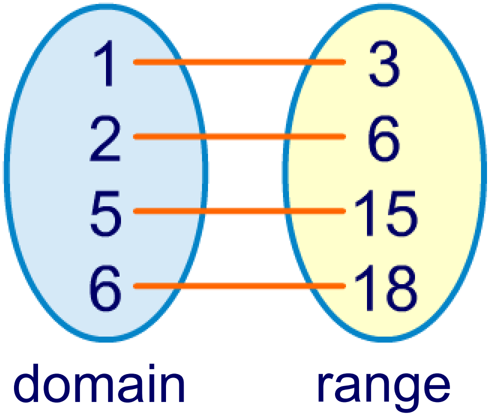
1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Reviewing functions

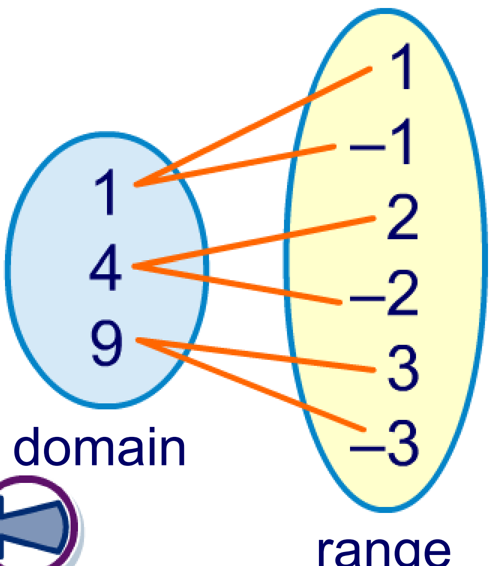
A **function** is a relation in which each element of the domain is mapped to *exactly one* element of the range.



function

$y = 3x$

In a function, each element of the domain is mapped to exactly one element of the range.



not a function

$y^2 = x$

In non-functions, elements of the domain may be mapped to more than one element of the range.

Vertical line test

Graphs of functions pass the **vertical line test** (**VLT**), meaning that if **every** vertical line in the plane intersects the graph in at most one point, then the graph is the graph of a function.

Press **start** to investigate graphs of different relations.

start

Review of composition of functions

The composition of function $g(x)$ with $f(x)$ is denoted

$$g \circ f(x) = g(f(x)).$$

It is important to consider the domain of $g(f(x))$ carefully.

If $f(x) = \sqrt{x}$ and $g(x) = x^2 - 4$, find A) $f \circ g(x)$ and B) $g \circ f(x)$. State the domain in each case.

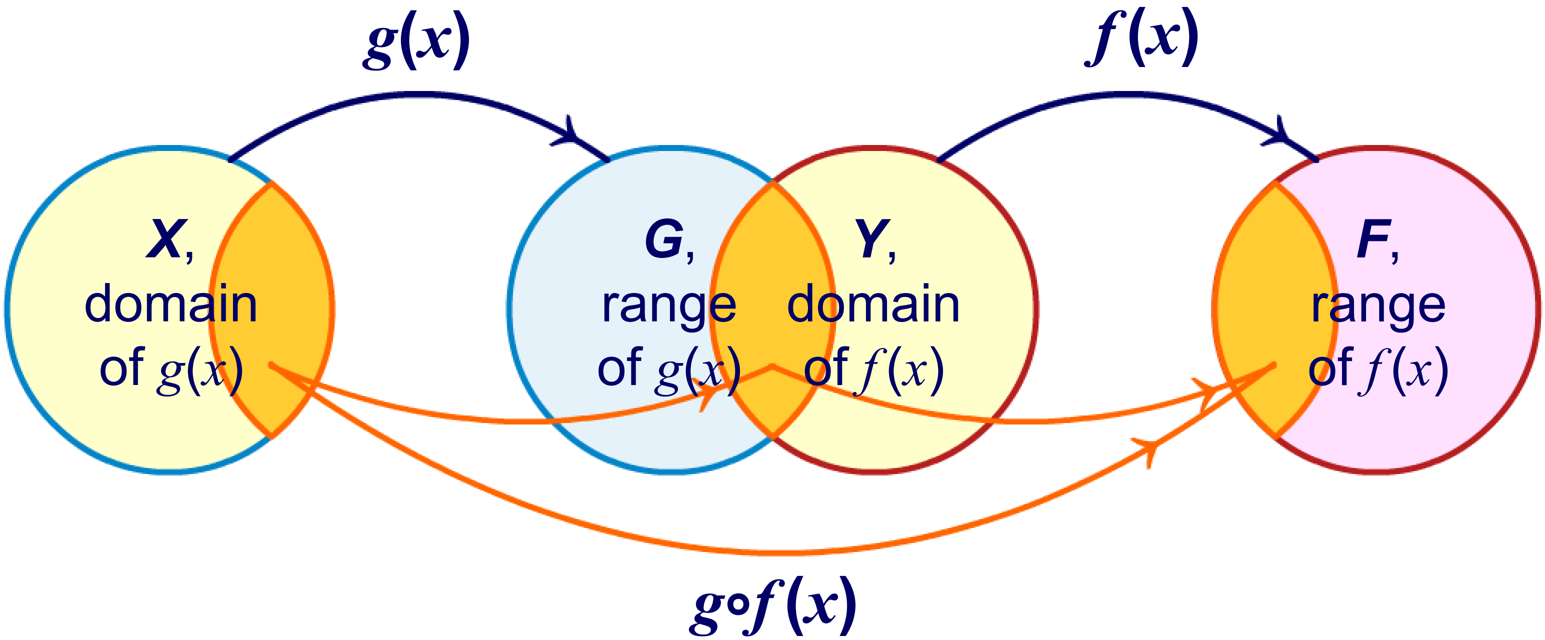
A) $f \circ g(x) = f(g(x)) = \sqrt{x^2 - 4}$

$x^2 - 4$ must be greater than or equal to zero to take its square root, which is true on the interval **$(-\infty, -2] \cup [2, \infty)$**.

B) $g \circ f(x) = g(f(x)) = (\sqrt{x})^2 - 4 = x - 4$

x must be greater than or equal to zero to take its square root. Subtracting 4 does not restrict the domain, so the domain for $g(f(x))$ is **$[0, \infty)$**.

Domain of composed functions



$g(x)$ takes input values from ***X*** to ***G*** and $f(x)$ maps ***Y*** to ***F***.

In order to compose $g(x)$ with $f(x)$, there must be overlap in the range of $g(x)$ and the domain of $f(x)$, i.e., $G \cap Y \neq \varnothing$.

The domain of $g∘f(x)$ is the set of values in the domain of $g(x)$ that map to the overlap in ***G*** and ***Y***.

Find the inverse of a function

Inverse functions "undo" each other; for example, squaring and square rooting are inverses.

To find the inverse of a function, interchange the x and y, then solve for y. Also remember to swap the domain and range.

If $f(x) = 5x - 4$ with domain and range of all real numbers, find $f^{-1}(x)$.

replace $f(x)$ with y:	$y = 5x - 4$
interchange the x and y:	$x = 5y - 4$
solve for y:	$(x + 4)/5 = y$
rewrite using $f^{-1}(x)$:	$\mathbf{f^{-1}(x) = (x + 4)/5}$ which also has domain and range all real numbers.

Compositions of inverse functions

What are the compositions $g \circ f(x)$ and $f \circ g(x)$ of $f(x) = \sqrt{x}$ and $g(x) = x^2$? What is true about these functions?

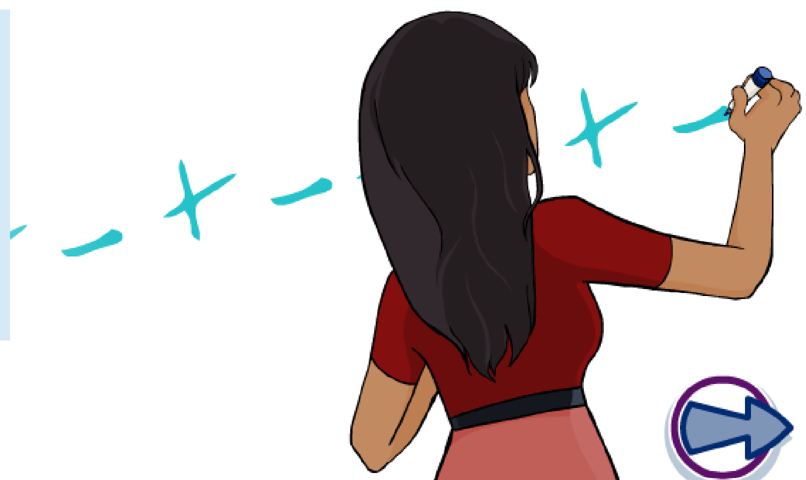
$g \circ f(x) = g(f(x)) = (\sqrt{x})^2 = x$, domain $[0, \infty)$

$f \circ g(x) = f(g(x)) = \sqrt{x^2} = x$, domain $[0, \infty)$

$f(x)$ and $g(x)$ are inverses.

If $f(x)$ and $g(x)$ are **inverses** of each other, then **$f \circ g(x) = g \circ f(x) = x$** for every x in the domains of $f(x)$ and $g(x)$.

Use composition of functions to prove that $f(x) = 5x - 4$ and $g(x) = (x + 4)/5$ are inverses of each other.



Domain and range of inverses

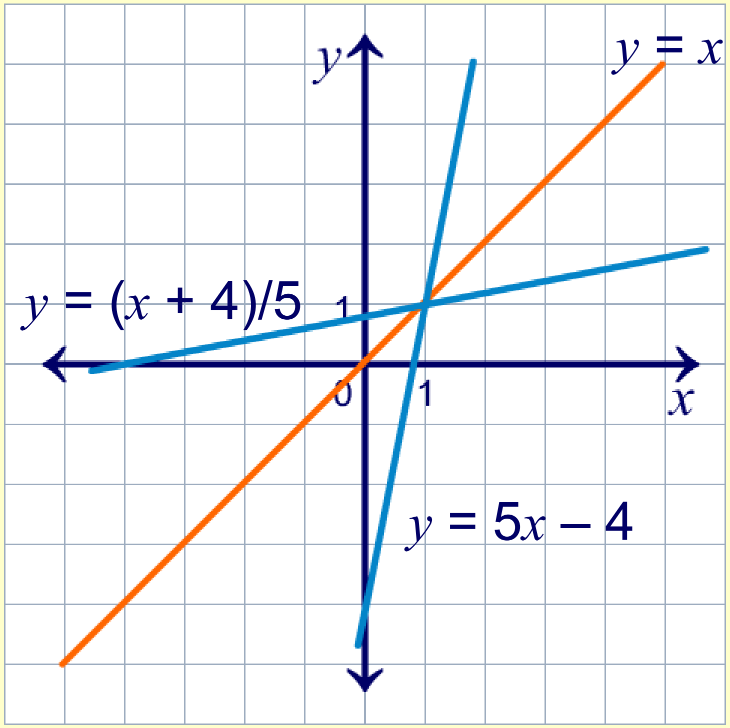
Complete the tables of values shown for the functions $f(x) = 5x - 4$ and $f^{-1}(x) = (x+4)/5$. What relationship is there between the domains and ranges?

x	$f(x)$
1	1
2	6
3	11
4	16
5	21

x	$f^{-1}(x)$
1	1
6	2
11	3
16	4
21	5

The domain and range are interchanged between $f(x)$ and $f^{-1}(x)$.

Sketch the graph of $f(x) = 5x - 4$ and $f^{-1}(x) = (x + 4)/5$ on the same axes along with the graph of $y = x$.



A function and its inverse are symmetric about the line $y = x$.

Reflecting a function in the line $y = x$ is equivalent to swapping its domain and range.

The horizontal line test

For a function to have an inverse that is also a function it must be **one-to-one**, meaning that for each x there is one and only one y, and vice versa.

The **horizontal line test** (**HLT**) says that if **every** horizontal line intersects the graph in at most one point, then the graph has an inverse.

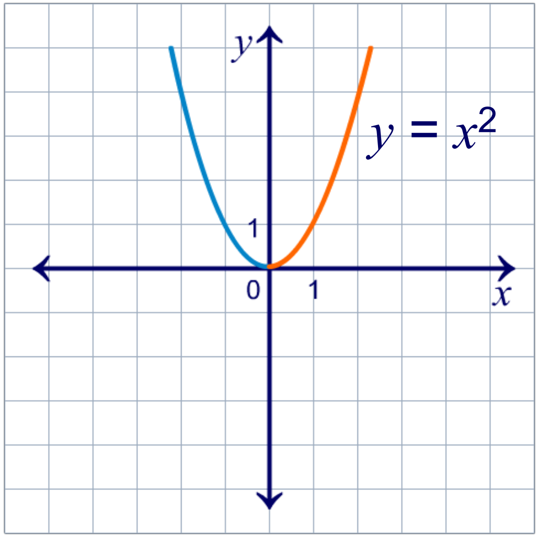
Press **start** to investigate graphs of different functions.

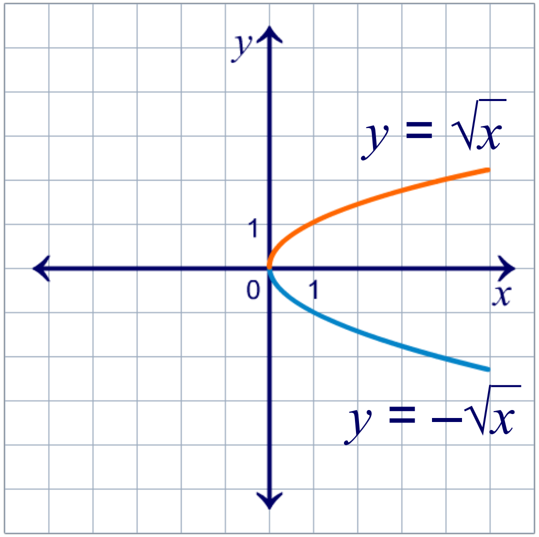
start

Restricting the domain

An important function that does not pass the horizontal line test is $f(x) = x^2$. It does not have an inverse that is a function.

However, if the domain is restricted to $x \geq 0$, $f(x) = x^2$ is one-to-one and therefore it has an inverse, $f^{-1}(x) = \sqrt{x}$.





The restriction is necessary because the domain and range are swapped when taking inverses.

Steps for finding the inverse of a function:

1. Graph $f(x)$ to determine if it has an inverse that is a function. If it is does not, state any necessary restrictions on the domain so that it has an inverse function.
2. Rewrite $f(x)$ as y.
3. Interchange the x and y.
4. Solve for y.
5. Rewrite y as $f^{-1}(x)$. State any restrictions on the domain of $f^{-1}(x)$.

Inverse function practice

	domain and range of $f(x)$	$f^{-1}(x)$	domain and range of $f^{-1}(x)$
$f(x) = \frac{2}{x-3}$	?	?	?
$g(x) = \sqrt[3]{x+2}$	?	?	?
$h(x) = \sqrt{x+2}$	?	?	?
$m(x) = x^3 + 8$	?	?	?

Inverse problem

A model describes a mathematical relationship between two or more quantities. The constants or parameters in the model are usually assumed to be known; however, sometimes they are not known. With measurements, the parameters can be estimated. This is called **inverse modeling**.

Press **start** to begin investigating inverse modeling for a pendulum.

start