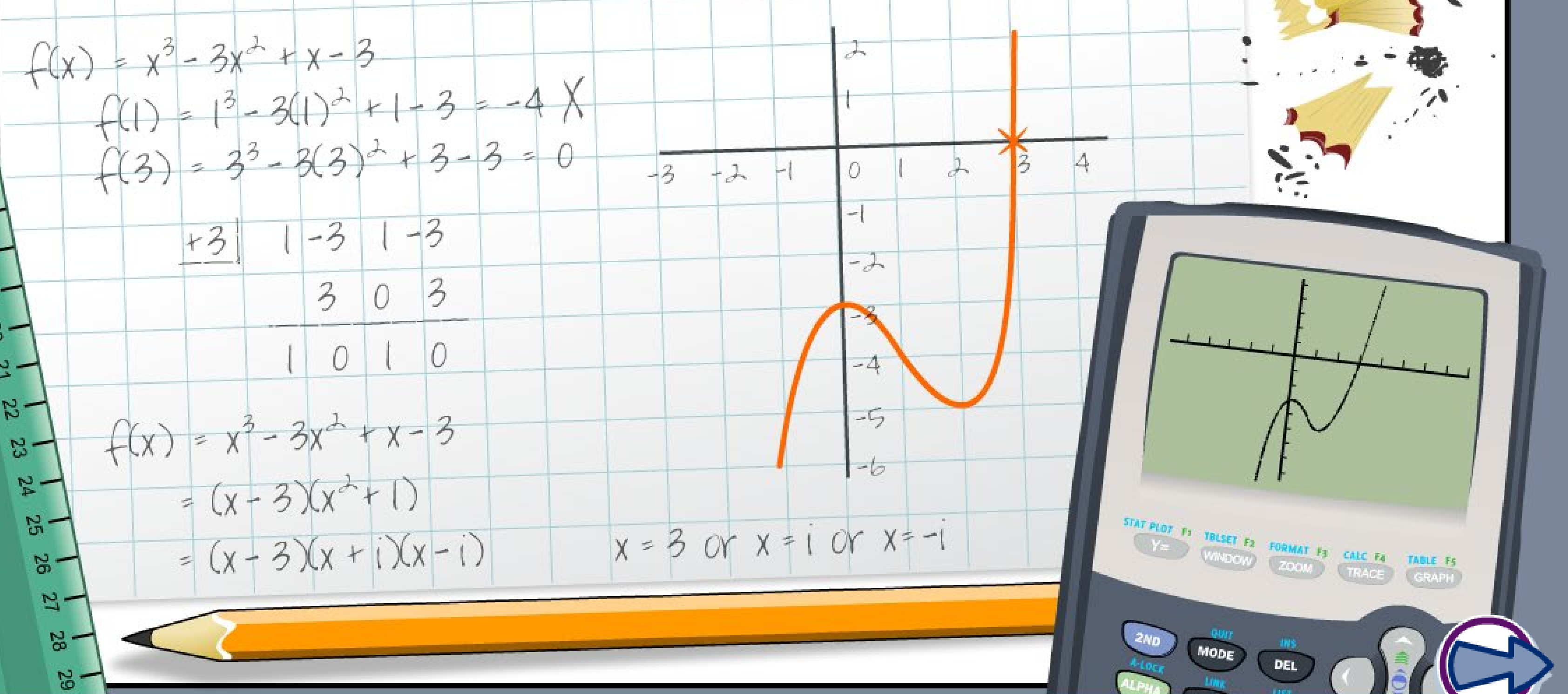
Normal Distribution



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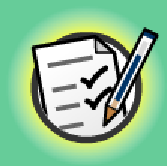
Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

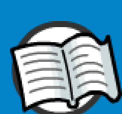
5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

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Random variables

A **random variable**, x, is defined as a variable whose values are determined by chance, such as the outcome of rolling a die.

A **continuous variable** is a variable that can assume any values in an interval between any two given values.

For example, height is a continuous variable. A person's height may theoretically be any number greater than zero.

What are other example of continuous variables?

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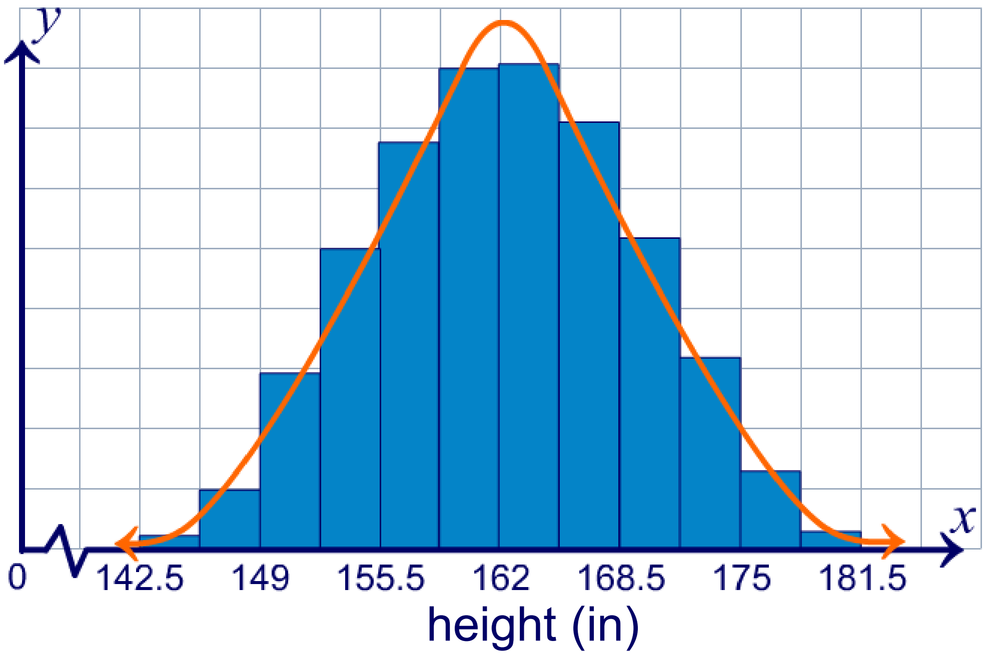
The histogram shows the heights of a sample of American women.



The histogram is a symmetrical bell-shape. Distributions with this shape are called **normal distributions**.

A curve drawn through the top of the bars approximates a **normal curve**.

In an ideal normal distribution the ends would continue infinitely in either direction. However, in most real distributions there is an upper and lower limit to the data.

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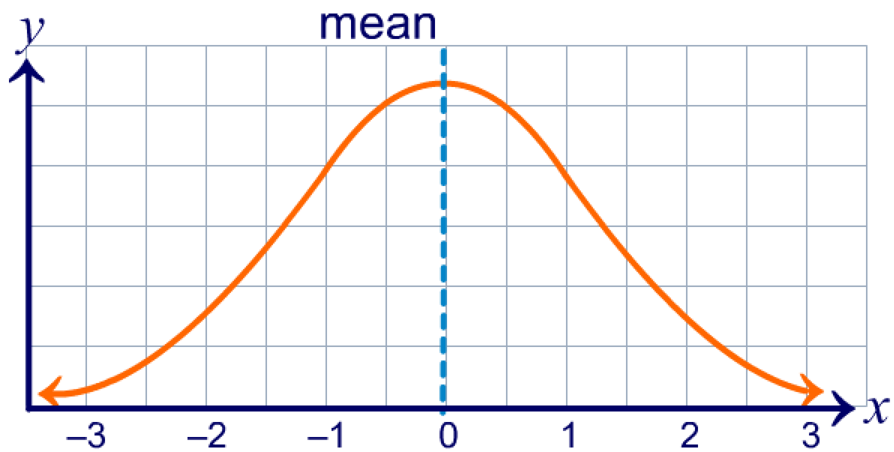
Mean and standard deviation

A normal distribution is defined by its **mean** and **variance**. These are parameters of the distribution.



When the mean is 0 and the standard deviation is 1, this is called the **standard normal distribution**.

the normal distribution: $x \sim N(\mu, \sigma^2)$

The random variable, x, has a normal distribution of mean, μ, and variance, σ^2.

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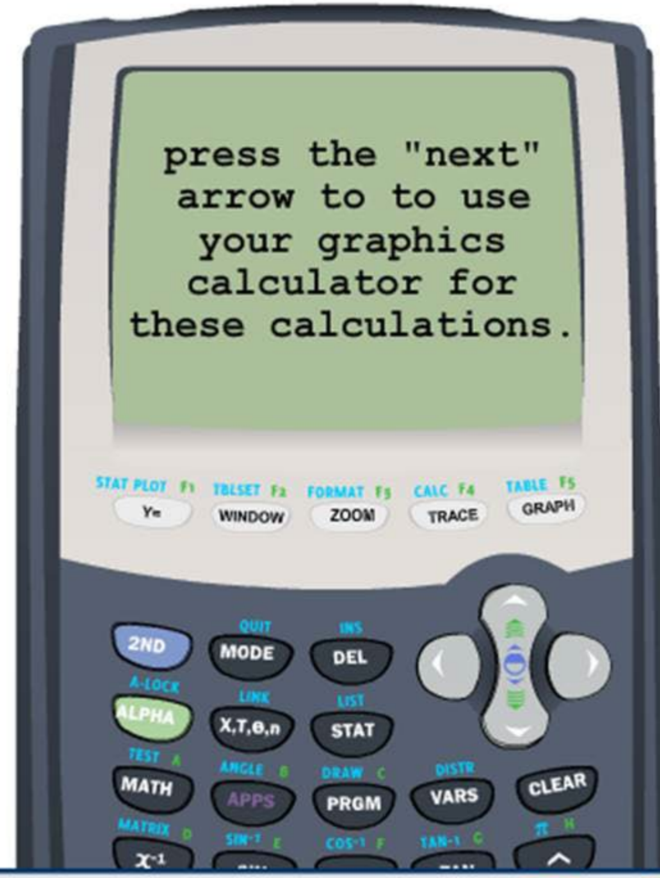
Using a graphing calculator to find μ, σ, and σ^2

You can quickly find the mean, variance, and standard deviation of a set of data using your calculator.

Press the next arrow to see how.



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The shape of a normal curve

All normal curves are bell-shaped, but the exact shape depends on the mean, μ, and the standard deviation, σ.

A small standard deviation indicates that the data is closely grouped to the mean. A large standard deviation indicates that the data is more spread out.

Press **start** to investigate the shape and location of the normal curve with different means and standard deviations.

start

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Continuous distribution

The normal distribution is a **continuous distribution**.

In a continuous distribution, the probability that a random variable will assume a particular value is zero. Explain why.

For a discrete random variable, as the number of possible outcomes increases, the probability of the random variable being one particular outcome decreases.

A continuous variable may take on infinitely many values, so the probability of each particular value is zero.

This means that the probability of the random variable falling within a range of values must be calculated, instead of the probability of it being one particular value.

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Area under the curve

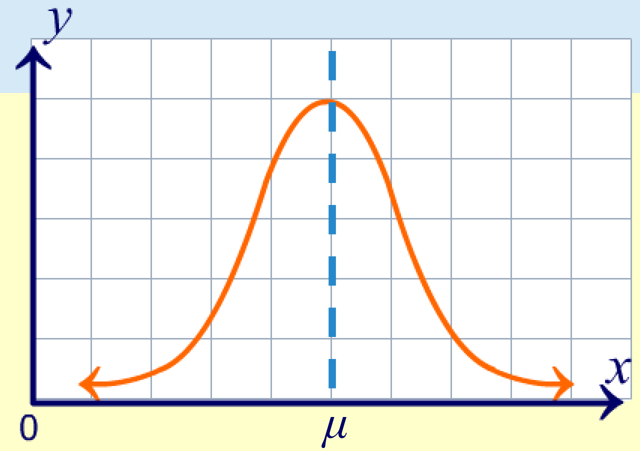
Since all probabilities must fall between 0 and 1 inclusive, the area under the normal distribution curve represents the entire sample space, thus it is equivalent to 100% or 1.

The probability that a random variable will lie between any two values in the distribution is equal to the area under the curve between those two values.

What is the probability that a random variable will be between μ and positive ∞?

The mean divides the data in half. If the area under the curve is 1.00 then the area to one side of the mean is:

$1 \times 0.5 = \mathbf{0.5}$



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Normal distribution curve

Drag the arrows to highlight a section of the normal distribution and press the yellow box to reveal the probability. Change the values of the mean and standard deviation for a new distribution.

What is the probability within one standard deviation of the mean? Two standard deviations? Three?

Press **start** to begin.

start

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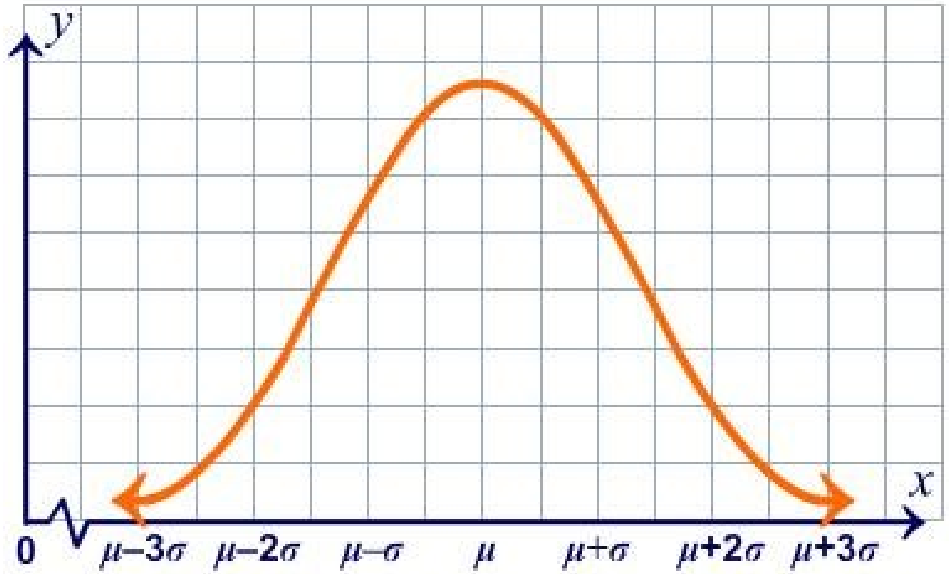
Properties of the normal distribution

one standard deviation

two standard deviations

three standard deviations

Press on the yellow buttons to see an example.



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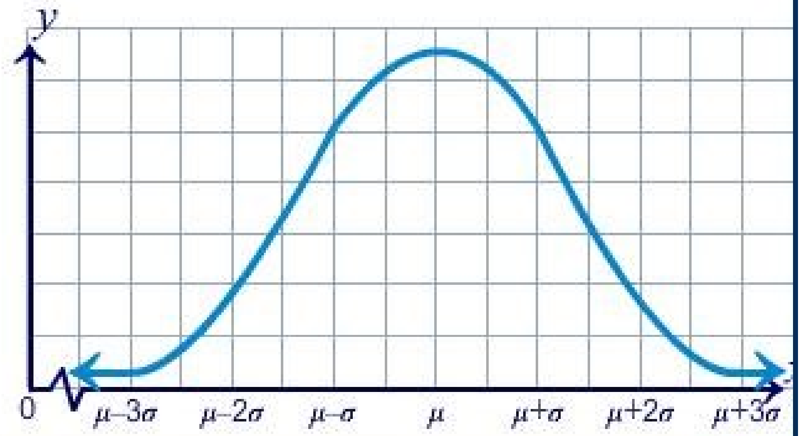
Heights of women in America

The heights of a sample of women in America follow a normal distribution.

The mean height is 162 cm and the standard deviation is 6.5 cm.



1) Draw a normal distribution graph of the data.

2) What percentage of women are shorter than 155.5 cm?

3) What percentage of women are taller than 181.5 cm?

4) What proportion of women are between 155.5 cm and 175 cm?

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Using a graphing calculator to find probability

The running times of a race have a mean of 63 minutes and a standard deviation of 3 minutes.

Use a graphing calculator to find the probability that a runner finished between 66 and 70 minutes.

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Applications of normal distributions

There are many situations where data follows a normal distribution, such as height and IQ.

Some data does not appear to fit the normal curve. However, if many samples are taken from this data, the mean of the samples tends towards a normal distribution.

This phenomenon is called the **central limit theorem**.

the central limit theorem:
Regardless of the distribution of a population, the distribution of the sample mean will tend towards a normal distribution as the sample size increases.

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Z-scores

Each normally distributed variable has a mean and standard deviation, so the shape and location of these curves vary.

All normally distributed variables can be transformed into a standard normally distributed variable using the formula for the standard score, or the ***z*-score**.

***z*-score:**

$$z = \frac{x - \mu}{\sigma}$$

The z-score transforms any normal distribution into a **standard normal distribution**, with a mean of 0 and a standard deviation of 1.

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Standard normal value

The z-score tells how many standard deviations any variable is from the mean.

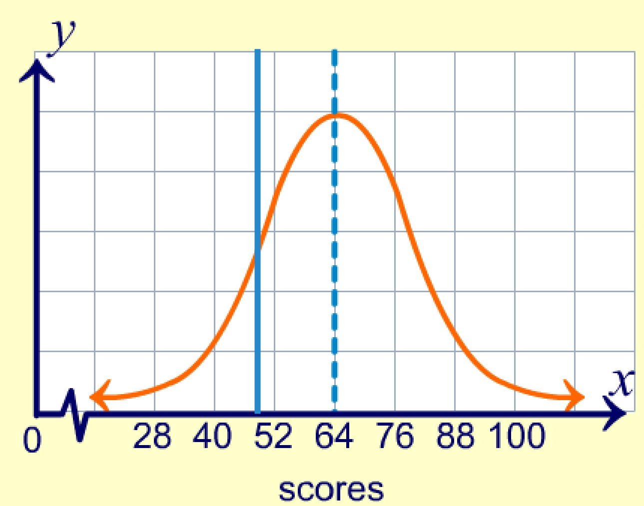
If $x \sim N(64, 144)$ represents the scores of an English test, find the z-score for a grade of 49.

find the standard deviation: $\sqrt{144} = 12$

write the z-score formula: $z = \dfrac{(x - \mu)}{\sigma}$

substitute in $x = 49$, $\mu = 64$ and $\sigma = 12$: $z = \dfrac{(49 - 64)}{12}$

solve for the z-score: $z = \mathbf{-1.25}$



The z-score means the grade is 1.25 standard deviations to the left of, or below, the mean.

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Standard normal distribution table

The area underneath the curve between 0 and the z-score can be found using the **standard normal distribution table**.

The rows show the whole number and tenths place of the z-score and the columns show the hundredths place.

Find the area between 0 and a z-score of 0.32.

Go down to the "0.3" row.

Follow the row across to the "0.02" column.

The area between 0 and 0.32 deviations is **0.1255**.

z	0.00	0.01	0.02	0.03
0.0	0.0000	0.0040	0.0080	0.0120
0.1	0.0398	0.0438	0.0478	0.0517
0.2	0.0793	0.0832	0.0871	0.0910
0.3	0.1179	0.1217	**0.1255**	0.1293
0.4	0.1554	0.1591	0.1628	0.1664

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Finding values using the z-score

If $x \sim N(20, 9)$ represents the amount of time people report spending on electronic devices per week, what values are 2.3 standard deviations away from the mean?

state the formula for the z-score: $z\text{-score} = (x - \mu) / \sigma$

find the standard deviation: $\sqrt{9} = 3$

enter a z-score of **–2.3** to find one standard deviation to the **left**:

$-2.3 = (x - 20) / 3$

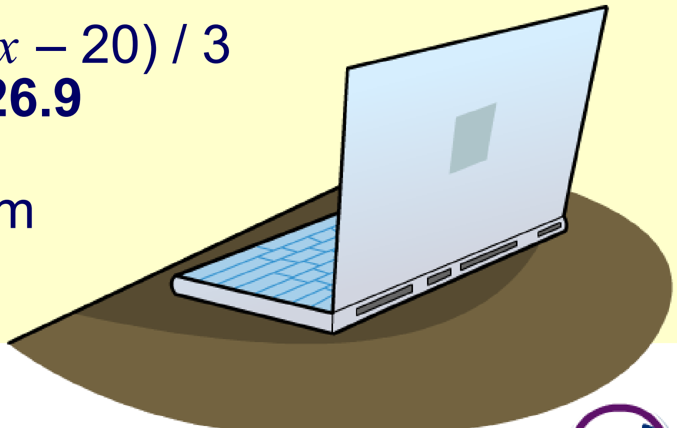
$x =$ **13.1**

enter a z-score of **2.3** to find one standard deviation to the **right**:

$2.3 = (x - 20) / 3$

$x =$ **26.9**

The values that are one standard deviation from the mean are **13.1** hours and **26.9** hours.



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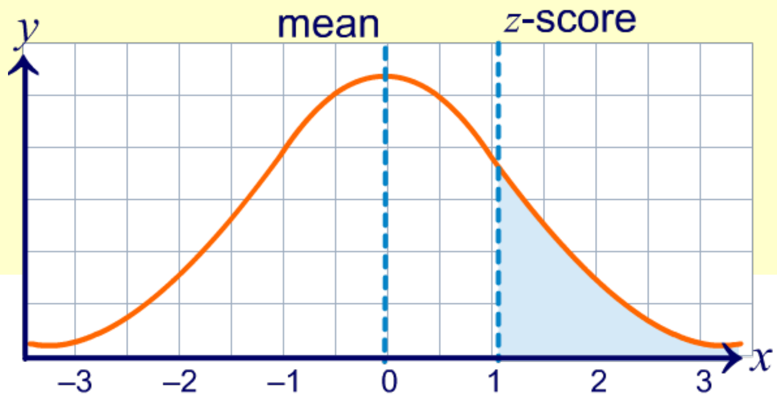
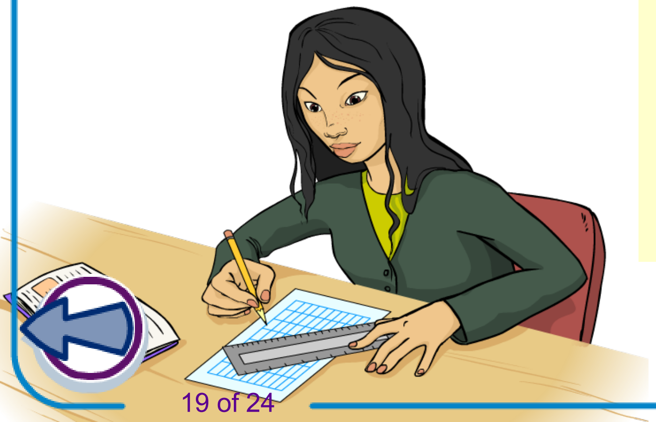
How would you find the area underneath the curve between the z-score and positive infinity?

Subtract the area between 0 and the z-score from 0.5 (the area from the mean to infinity).



How would you find the area between two z-scores?



For z-scores on the same side of the mean, subtract the smaller area from the larger area.

For z-scores on the opposite sides of the mean, add the areas.

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Test scores

Sarah scores 82% on her math test. The class scores are normally distributed with a mean of 70 and standard deviation of 10. Is she in the top 10th percentile?

Press the "=" button to show the calculations step by step.

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Distribution of scores

The table shows the number of students in two classes who got each score on their history test.

1) Draw a histogram for the results from each class.

2) Does the histogram approximate a normal curve?

class 1 class 2

Click on the yellow buttons to see the answers.

score	class 1	class 2
1	5	1
2	7	3
3	3	4
4	2	5
5	2	5
6	5	3
7	5	2
8	6	1

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Checking for normality

The number of calories contained in a selection of fast-food sandwiches are shown here. Is the data noramlly distributed?

390	560	225
405	535	720
580	660	470
300	530	390
320	290	675
540	440	530

histogram

Pearson's Index

Press the yellow buttons to learn how to check for normality using each method.

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Practice using Pearson's Index

A 500 g bag of flour usually weighs slightly more or less than 500 g. The weights of 15 bags are shown. Use Pearson's Index to decide if the data is skewed.

weight (g)
498
500
505
502
500
501
500
497
498
499
495
503
502
496
504

use your graphing calculator to find the mean, median and standard deviation:

$\mu = 500$

$\sigma = 2.8$

median = 500

substitute into the formula for Pearson's Index:

$$PI = 3(\mu - \text{median}) / \sigma$$
$$= 3(500 - 500) / 2.8$$
$$= 0$$

analyze: A Pearson's Index of 0 indicates that the data is **not skewed**.

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Normal distribution quiz

Normal distribution quiz

Question: 1/4

The weights of lemurs at a zoo have a mean of 5 lbs and standard deviation of 0.75 lbs. What is the z-score for a weight of 4.1 lbs?

0.93	–1.20	0.71	–0.7

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