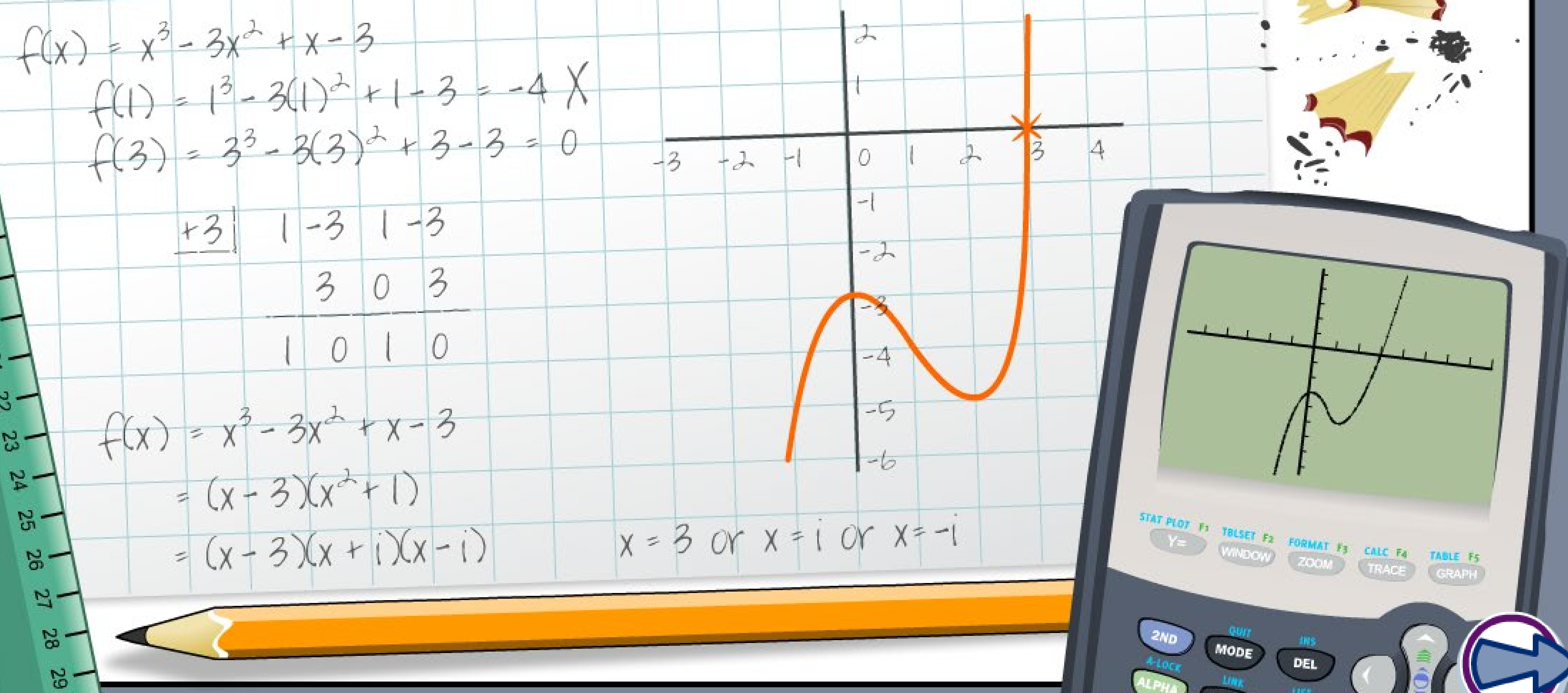
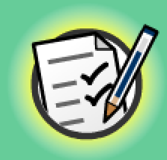
Probability Distributions

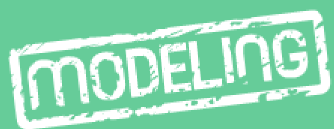


Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

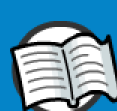
This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

Random variables

A **random variable**, x, is a variable whose values are determined by chance, such as the outcome of rolling a die.

A **discrete random variable** is a random variable that can only take on a countable number of values, such as the integers, or a set of whole numbers.

e.g. {1, 2, 3, 4, 5, 6}

A **continuous random variable** is a random variable that can assume all values in an interval between any two given values, such as the real numbers, or an interval.

e.g. [0, 25]

What random variables relate to a ladybug population?

Continuous vs. discrete variables

Are the variables continuous or discrete?

continuous	discrete

time

S C ?

Suppose you flip a coin 20 times, and it lands heads up x times.

What type of variable is x? What are its possible values?

x is a **discrete random variable** that counts the number of successful **trials** in the **experiment** (success is landing heads up).

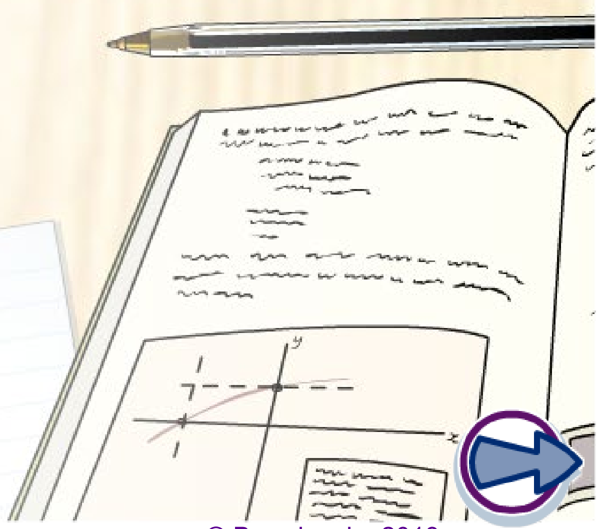
If you repeat this experiment, the value of x will likely change, but **$0 \leq x \leq 20$** and is a **whole number**.

x follows a **binomial distribution**.

Binomial distribution

The characteristics of a **binomial distribution** are:

- two possible outcomes on each trial: success (S) and failure (F)
- the trials are independent of each other
- the probability of S is constant between trials: $P(S) = p$
- F is the complement of S: $P(F) = 1 – P(S) = 1 – p = q$
- the binomial random variable x is the number of successes in the n trials.



In general, a distribution is described by **parameters**.
Parameters are known quantities of the distribution.

What parameters describe a binomial distribution?

- number of trials in the experiment, n
- probability of success, p

$x \sim B(n, p)$ is common notation to say a random variable x follows a binomial distribution, with n trials and probability of success p.

Using this notation, write the distributions of:

1) a single fair coin landing heads up

2) the total number of heads after 10 flips of a coin that lands heads up 60% of the time.

1) $x \sim B(1, 0.5)$ **2)** $x \sim B(10, 0.6)$

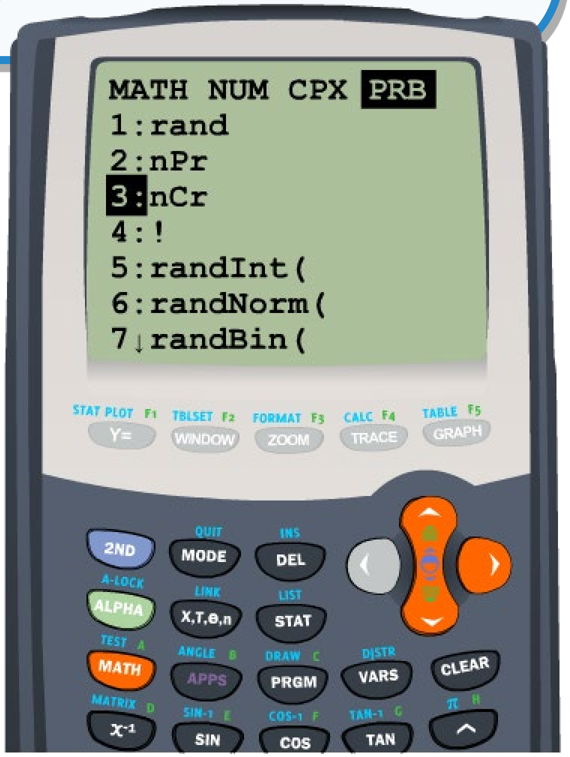
probability of a binomial experiment:

$$b(k; n, p) = {}_nC_k\, p^k q^{n-k}$$

where n is the number of trials, k is the number of successes, p is the probability of success, and $q = 1 - p$ is the probability of failure.

Remember that ${}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$

nCr is found on the “MATH” “PRB” menu on a graphing calculator.



Practice

A manufacturer determines that 5% of the computer chips produced are defective. What is the probability that a batch of 25 will have exactly 3 defective?

Let x be the number of defective chips out of 25 chips produced.
Use the formula for the probability of a binomial experiment:

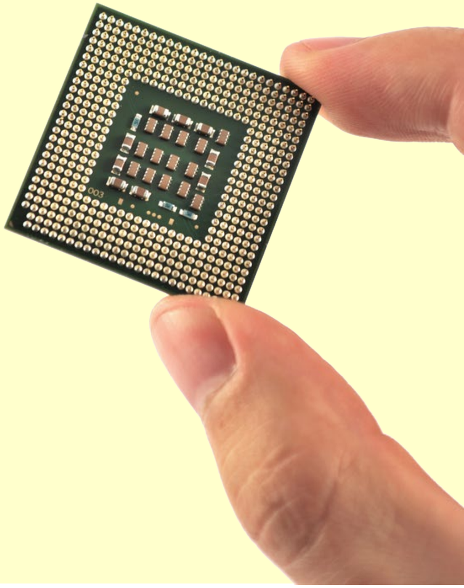
$$P(x = 3) = b(3;\, 25,\, 0.05)\ (= {}_nC_k\, p^k\, q^{n-k})$$

$$= {}_{25}C_3(0.05)^3(0.95)^{22}$$

$$= (2300)(0.000125)(0.324)$$

$$= 0.0930 \text{ (to the nearest thousandth)}$$

The probability of exactly 3 defectives is **0.0930**.



Using binompdf(

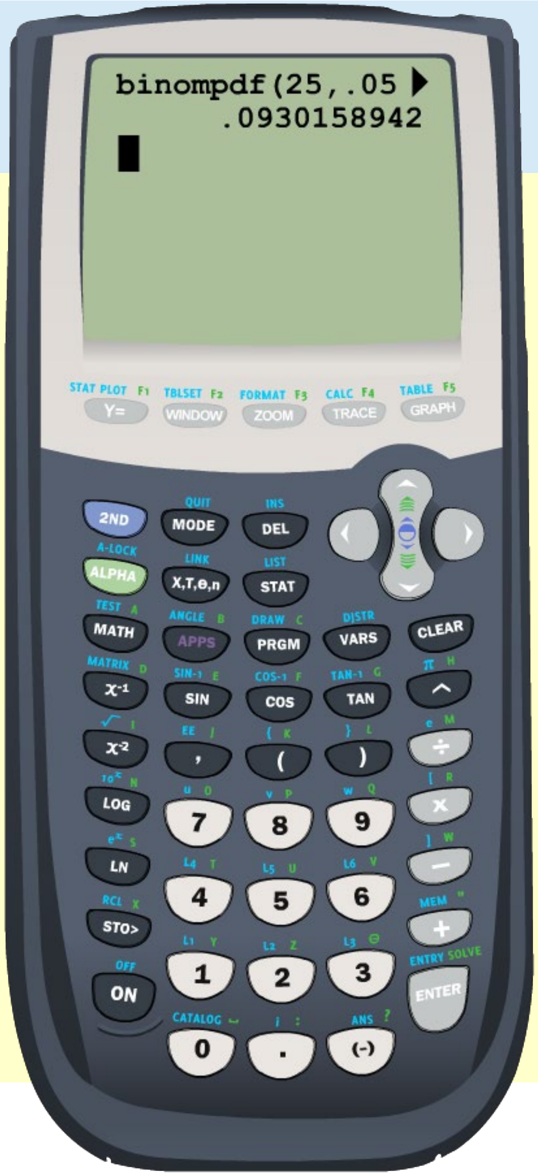
How do you find $b(3; 25, 0.05)$ directly using a graphing calculator?

- Press “2ND” “VARS” to get to the “DISTR” menu.

- Scroll down to “binompdf(” and press “ENTER”, then key in the variables.

  n = 25, number of trials
  p = 0.05, probability of success
  x = 3, number of successes

- Scroll to paste and press “ENTER” and press “ENTER” again to calculate.



A manufacturer determines that 5% of the computer chips produced are defective. What is the probability that a batch of 25 will have no more than 2 defective?

Let x be the number of defective chips out of 25 chips produced.

$P(x \leq 2)$ = $P(x = 0 \text{ or } 1 \text{ or } 2)$

= $P(x = 0) + P(x = 1) + P(x = 2)$

= $b(0; 25, 0.05) + b(1; 25, 0.05) + b(2; 25, 0.05)$

= ${}_{25}C_0(0.05)^0(0.95)^{25} + {}_{25}C_1(0.05)^1(0.95)^{24} + {}_{25}C_2(0.05)^2(0.95)^{23}$

= 0.277 + 0.365 + 0.231

= 0.873 (to the nearest thousandth)

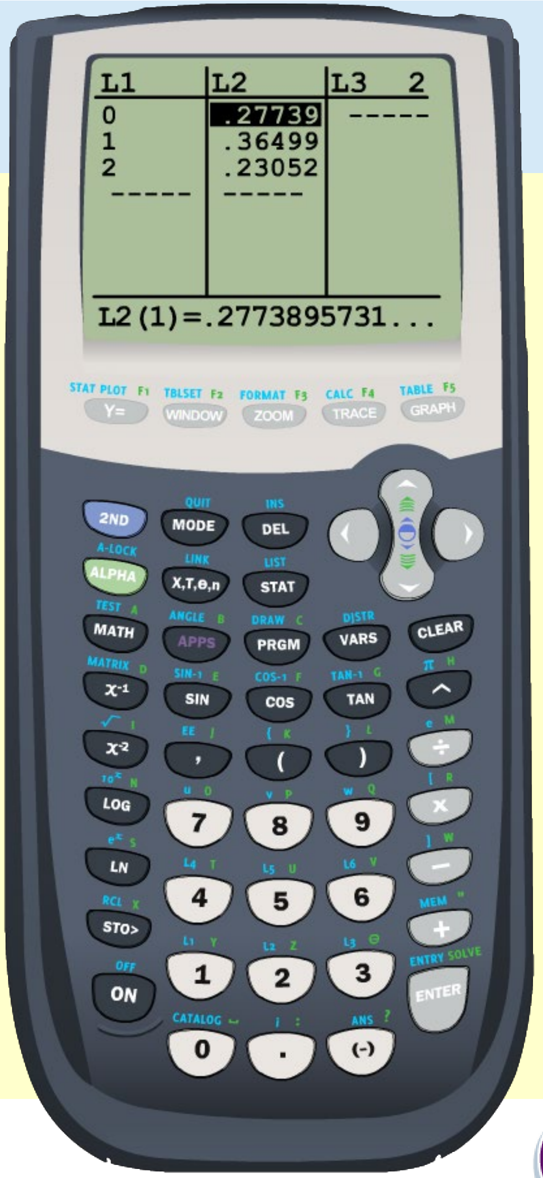
The probability of no more than 2 defective chips is **0.873**.

Using lists to find probabilities

If $x \sim B(25, 0.05)$, how do you find $P(x \leq 2)$ using a graphing calculator?

- Use the “STAT” menu. Enter the numbers 0, 1, 2, in L1. Then select L2.
- Press “2ND” “VARS” for the “DISTR” menu. Scroll down to “binompdf(” and press “ENTER”.
- Fill in n = 25, p = 0.05 but leave x blank. Then select “paste” to populate L2 with the probability distribution.
- Add the probabilities of 0, 1, and 2.

The probability of no more than 2 defectives is about **0.873**.



Using binomcdf(

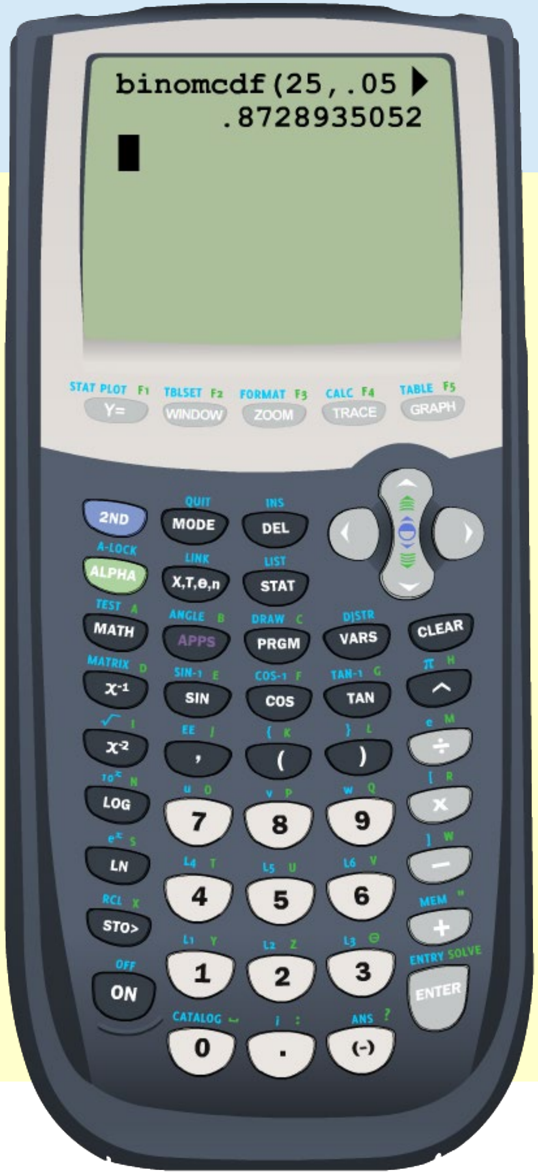
If $x \sim B(25, 0.05)$, how do you find $P(x \leq 2)$ using a graphing calculator?

- Press “2ND” “VARS” function to get to the “DISTR” menu.
- Scroll down to “binomcdf(” and press “ENTER”, then key in the variables.

  $n = 25$, $p = 0.05$, $x = 2$

- Scroll to paste and press “ENTER” and press “ENTER” again to calculate.

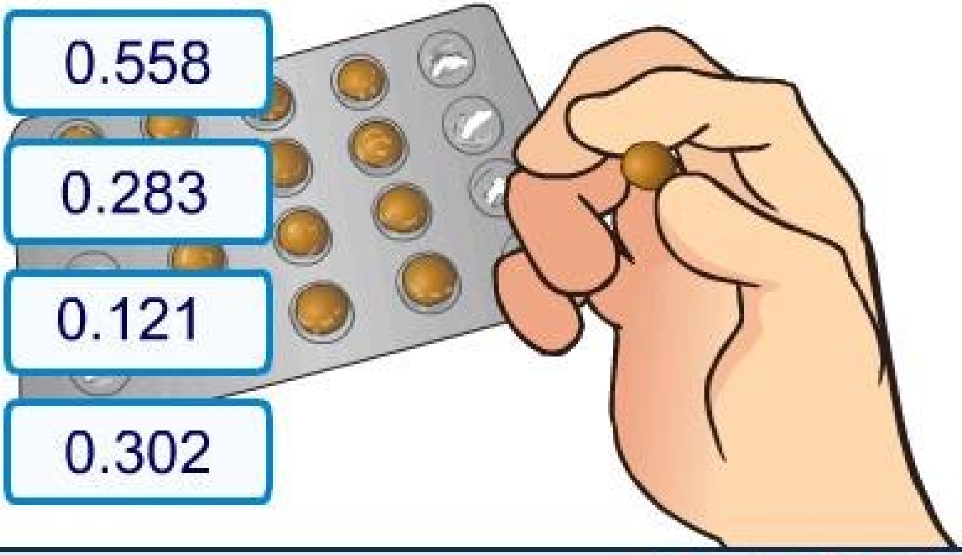
Verify that this is the same as
$b(0; 25, 0.05) + b(1; 25, 0.05) + b(2; 25, 0.05)$.



A medical company says that there is an 80% probability that those taking their headache medicine will get relief.

Match the number of people who feel relief to the probability based on the company's claim.

number of people	probability
10 out of 12	0.558
less than 7 out of 10	0.283
more than 9 out of 12	0.121
8 out of 10	0.302



Expected value

Distributions have a theoretical mean. This is also called the **distribution mean** or **expected value**.

For a binomial random variable, this is found by multiplying the number of trials (n) by the probability of success (p):

expected value for binomial random variable: $\mu = np$

What is the expected number of heads for
1) 1 flip of a fair coin, $x \sim B(1, 0.5)$
2) 10 flips of a biased coin, $x \sim B(10, 0.6)$.

1) $\mu = np = 1 \times 0.5 =$ **0.5**

2) $\mu = np = 10 \times 0.6 =$ **6**

Variance and standard deviation

Distributions have a **variance** (σ^2), which describes spread, i.e. the difference between the random variable and its mean.

Variance depends on the parameters of the distribution.

distribution variance for binomial random variable: $\sigma^2 = npq = np(1 - p)$

Standard deviation is the square root of variance (σ), and it describes the expected difference between the mean and the random variable.

distribution standard deviation for binomial random variable: $\sigma = \sqrt{npq} = \sqrt{np(1 - p)}$

Statistics

The parameters of the distribution might not be known.

For example, you might wish to determine the probability that a biased coin lands head up.

Here, you know the distribution is binomial, but you do not know p.

A **statistic** is a function of random variables.

Usually, a statistic tries to **estimate** the value of a parameter of a distribution.

The **sample mean** is an important statistic. It tries to estimate the mean of a distribution based on different trials.

sample mean formula: $$\bar{\mu} = \frac{\sum x_i}{n} = \frac{x_1 + x_2 + \ldots + x_{n-1} + x_n}{n}$$

- n is the number of trials
- x_i is the value of the random variable on trial i
- $\sum$ is the sum over all trials.

Sample mean is denoted by the Greek letter $\bar{\mu}$ with a bar over the top.

Bit errors in communication system

MODELING

A communication system sends data as a digital signal of bits: either 0 or 1. The digital signal is corrupted by noise, and the probability that the noise changes a bit from 0 to 1 (or 1 to 0) is 0.01.

What is the expected number of incorrect bits if 100 are sent? Model this problem using a binomial distribution.

- each bit is a trial and there are 100 bits, so $n = 100$
- the probability of an error is 0.01, so $p = 0.01$

model the total number of errors as a random variable: $x \sim B(100, 0.01)$

find the expected number of errors: $\mu = np = 100 \times 0.01 = \mathbf{1}$

Sample variance

Sample variance (s^2) and standard deviation (s) describe the difference between individuals and the sample mean.

sample variance:

$$s^2 = \frac{\sum(x_i - \bar{\mu})^2}{n}$$

sample standard deviation:

$$s = \frac{\sqrt{\sum(x_i - \bar{\mu})^2}}{\sqrt{n}}$$

- n is the number of trials
- x_i is the value of the random variable on trial i
- $\sum$ is the sum over all trials
- $\bar{\mu}$ is the sample mean.

Common discrete distributions

binomial | Poisson | geometric

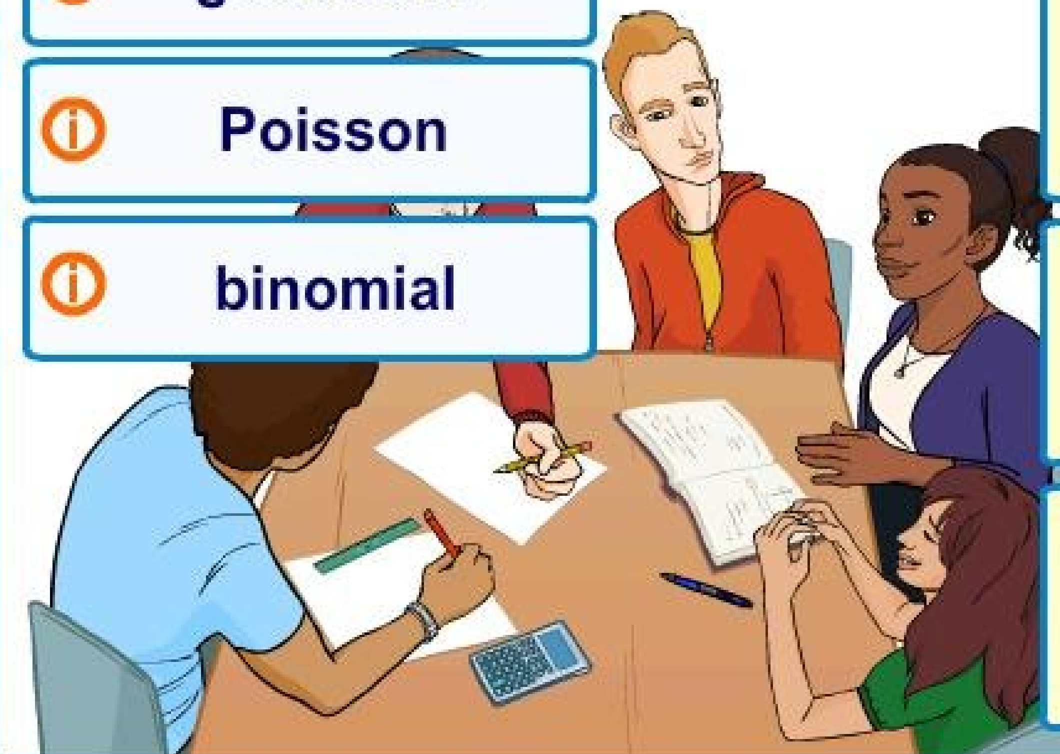
There are many types of discrete distributions. Each type models a different scenario.

Press the tabs to find out more about some of the more common types.

Modeling with distributions

Match each scenario to the distribution that best models it

- geometric
- Poisson
- binomial



- The number of correct answers when guessing randomly on a multiple choice test.
- The number of candidates interviewed for a job until a match is found.
- The number of car accidents at 34th and Walnut.