

Radical Functions

$$f(x) = x^3 - 3x^2 + x - 3$$

$$f(1) = 1^3 - 3(1)^2 + 1 - 3 = -4 \quad \times$$

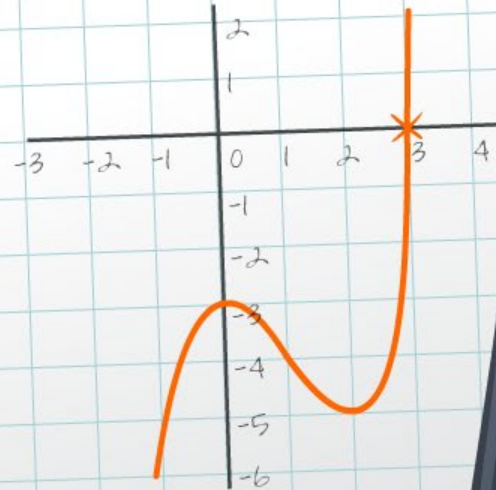
$$f(3) = 3^3 - 3(3)^2 + 3 - 3 = 0$$

$$\begin{array}{r|rrrr} +3 & 1 & -3 & 1 & -3 \\ & & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

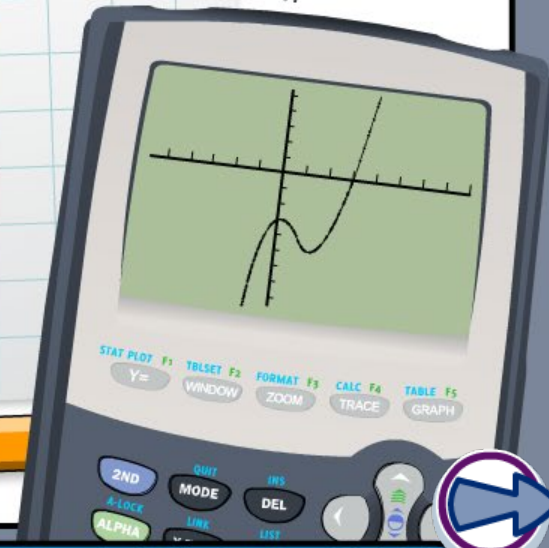
$$f(x) = x^3 - 3x^2 + x - 3$$

$$= (x - 3)(x^2 + 1)$$

$$= (x - 3)(x + i)(x - i)$$



$$x = 3 \text{ or } x = i \text{ or } x = -i$$



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.



This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

- 1) **Make sense of problems and persevere in solving them.**
- 2) **Reason abstractly and quantitatively.**
- 3) **Construct viable arguments and critique the reasoning of others.**
- 4) **Model with mathematics.**
- 5) **Use appropriate tools strategically.**
- 6) **Attend to precision.**
- 7) **Look for and make use of structure.**
- 8) **Look for and express regularity in repeated reasoning.**



This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.





Students are investigating how the speed of sound, v , varies with temperature, T . They have experimental data summarized in the table; however, it was not possible for them to collect data at very low temperatures. What they know is that theoretically sound cannot travel at absolute zero (-273°C).

$T (^{\circ}\text{C})$	v (m/s)	$u = v^2$
0	355	126,025
50	385	148,225
100	415	172,225
150	440	193,600
200	465	216,225
-273	0	0

The students plan to do a linear regression to find a relationship to describe the speed of sound. Why does their table include $u = v^2$?

They are planning to check if v varies linearly with T , **OR** if $u (= v^2)$ varies linearly with T . The second regression will check whether v depends on the square root of T .





Perform a linear regression using

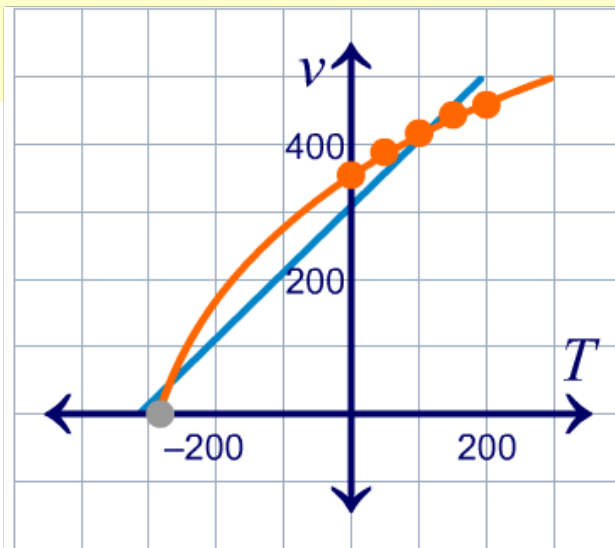
- T as the independent and v as the dependent variables
- T as the independent and $u = v^2$ as the dependent variables, then solve for v and plot the results.

$$v = 1.006T + 305.3$$

$$u = 457.8T + 125397$$

$$v^2 = 457.8T + 125397$$

$$v = 354.1\sqrt{1 + T/273.9}$$



Which gives better results?
How is the theoretical data point at absolute zero helpful in determining which is the better model?



Speed of sound experiment (3)

MODELING



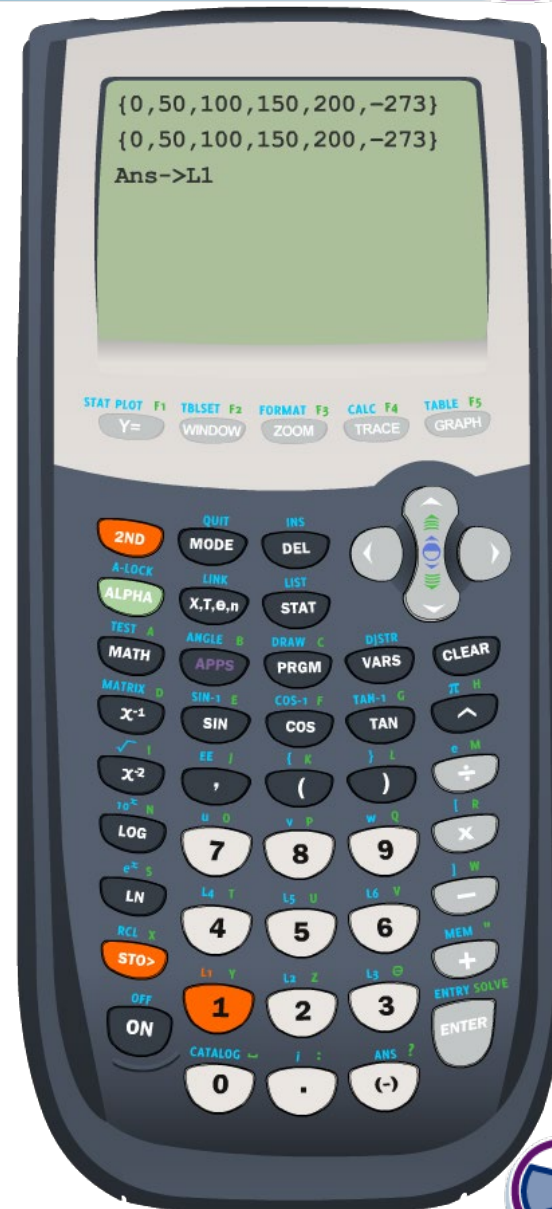
boardworks

Create a list for the temperature data using the “{” and “,” keys.

Store the list in L1 by pressing “STO>” then “2nd” “1”, then “ENTER”.

T (°C)	v (m/s)	$u = v^2$
0	355	126,025
50	385	148,225
100	415	172,225
150	440	193,600
200	465	216,225
-273	0	0

Similarly, store the data for v in L2, and the data for u in L3.



Speed of sound experiment (4) MODELING



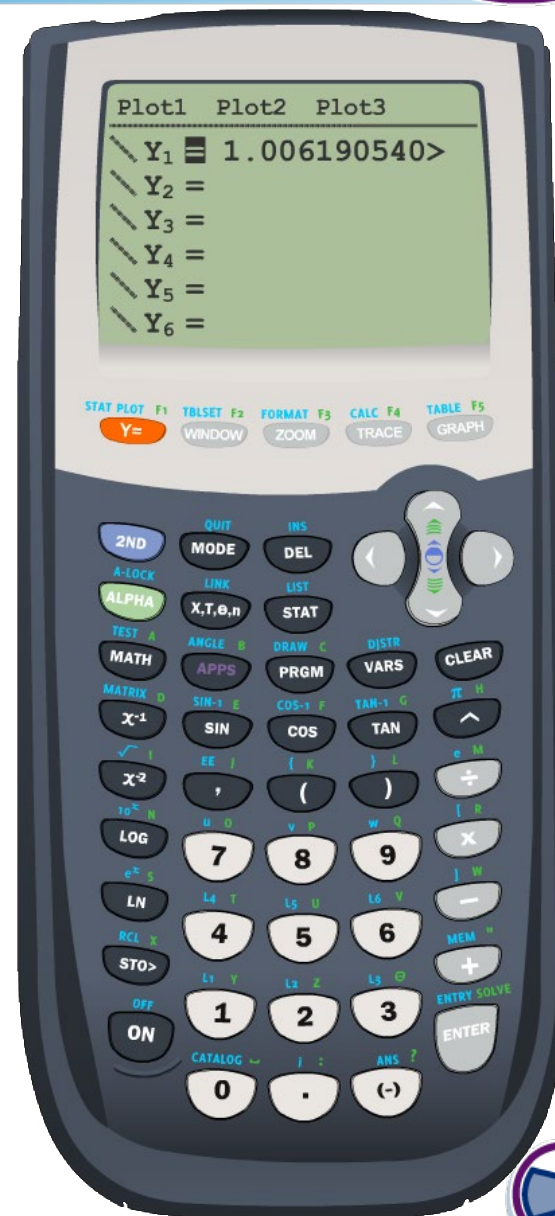
To perform a linear regression on a graphing calculator for L1 and L2, press the “STAT” button, scroll to the “CALC” menu and press “4” for $\text{LinReg}(ax+b)$.

Press “2nd” “1” to use L1 as the Xlist. Similarly, scroll to the Ylist field and press “2nd” “2” to use L2.

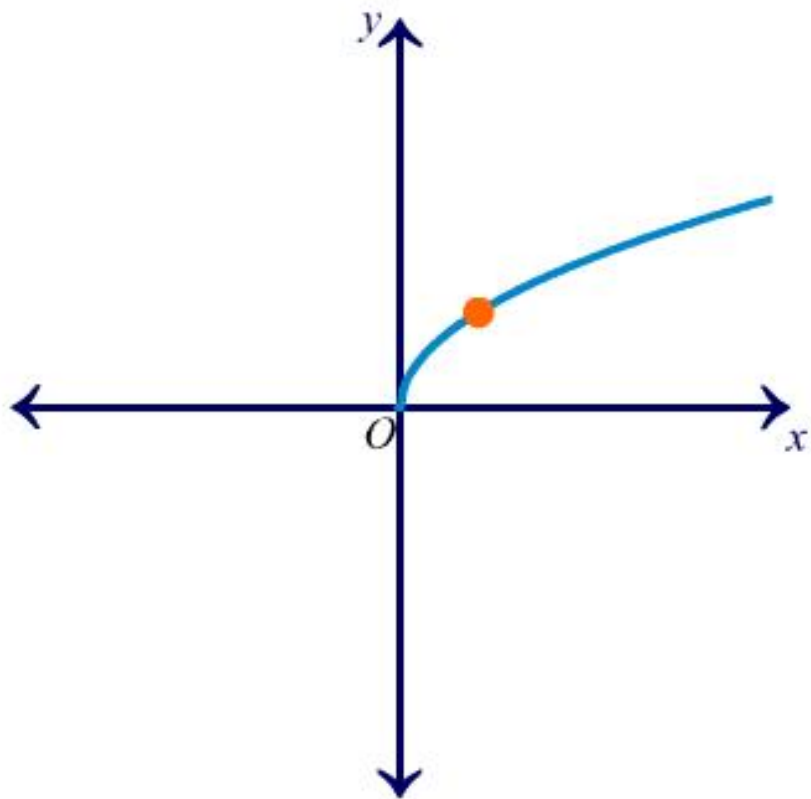
Scroll to the Store RegEQ field. Press “VARS”, scroll to “Y-VARS” menu and press “1” for Function. Press “1” to store the equation in the Y1 field in Y=.

Scroll to Calculate and press “ENTER”.

The result is also stored in Y=.



Exploring radical functions



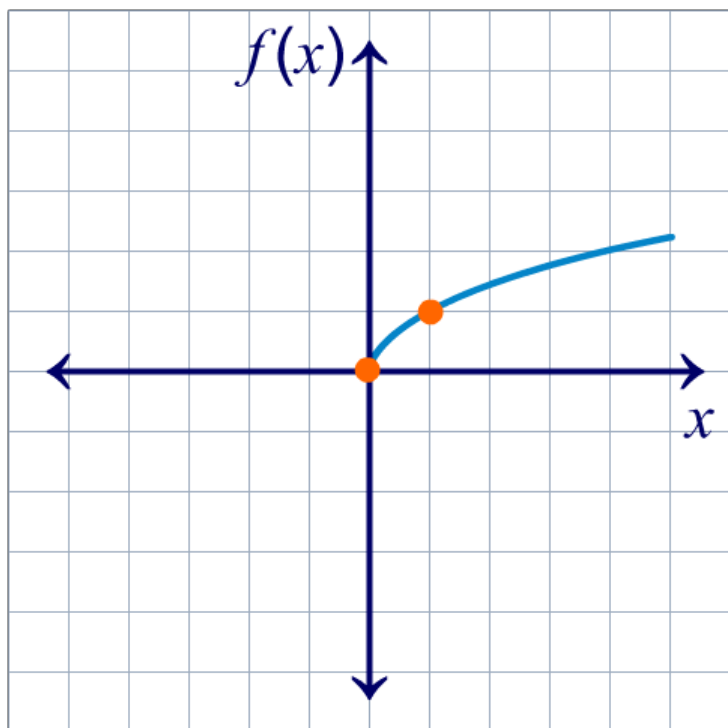
*Press the arrows
to change the index.*

$$y = \sqrt{x}$$





square root function: $f(x) = \sqrt{x}$



vertical asymptote: none

horizontal asymptote: none

domain: $[0, \infty)$

range: $[0, \infty)$

roots: $x = 0$

special points: $(1, 1)$

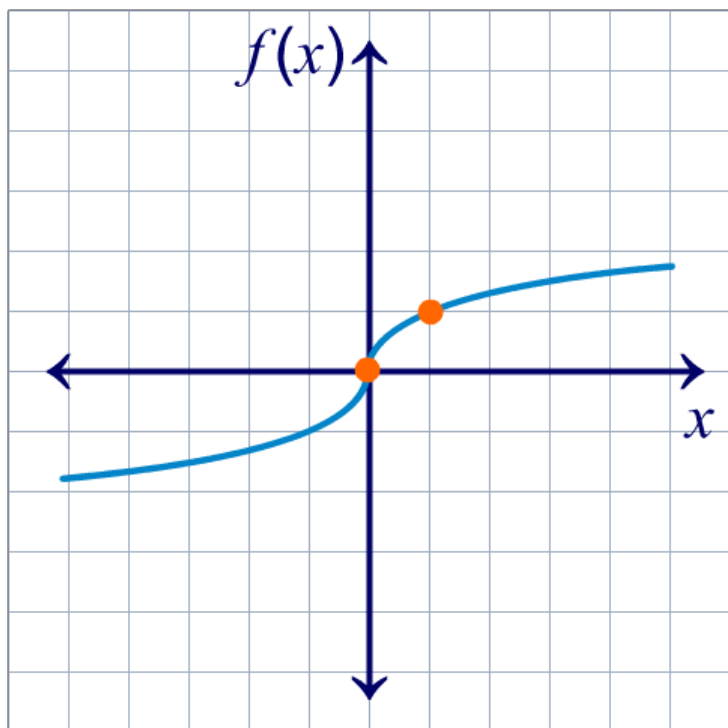
All functions of the form $\sqrt[n]{x}$ with an even index n have the same basic characteristics and shape.





cube root function:

$$f(x) = \sqrt[3]{x}$$



vertical asymptote: none

horizontal asymptote: none

domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

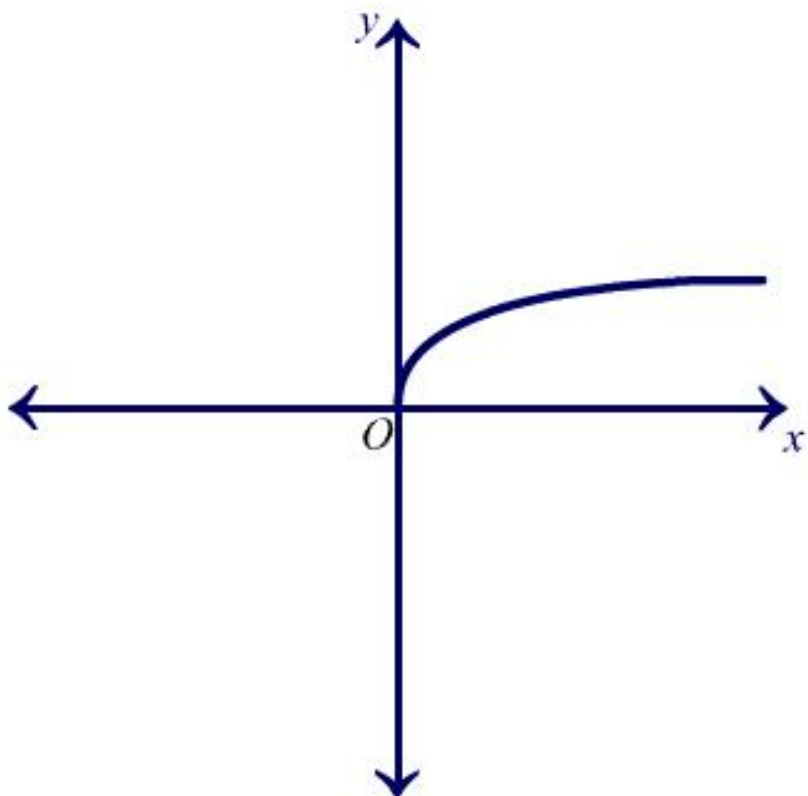
roots: $x = 0$

special points: $(1, 1)$

All functions of the form $\sqrt[n]{x}$ with an odd index n have the same basic characteristics and shape.



Reflecting radical functions



Press the arrows to change the index to 2 or 3.

$$— f(x) = \sqrt{x}$$

Press **show** or **hide** to show or hide the transformed functions.

$$— y = f(-x)$$

show

$$— y = -f(x)$$

show

$$— y = f^{-1}(-x)$$

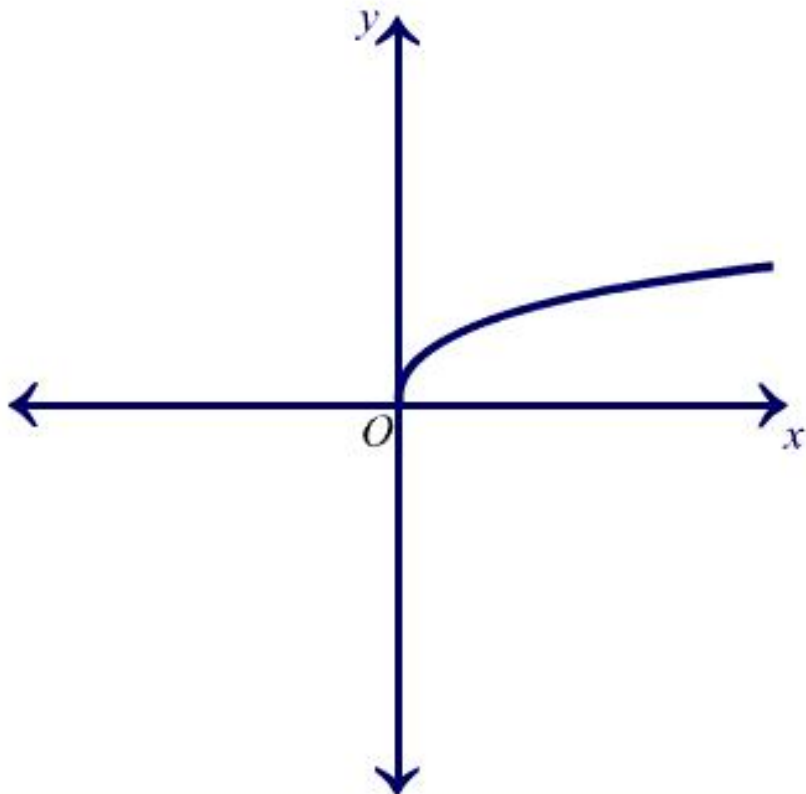
show



Translating radical functions

Press the arrows to change the index or transform the functions.

Press **show** or **hide** to show or hide the transformed functions.



— $f(x) = \sqrt{x}$

— $y = f(x) + 0$

— $y = f(x + 0)$

show

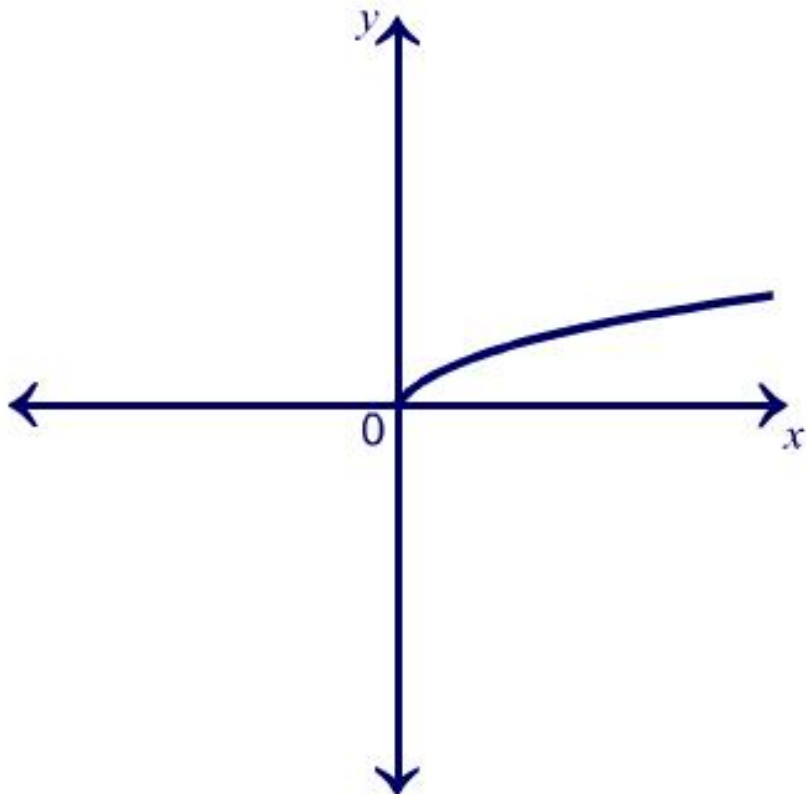
show





Stretching radical functions

Press the arrows to transform the function. Press **show** or **hide** to show or hide the transformed functions.



— $f(x) = \sqrt{x}$

— $y = 1f(x)$

— $y = f(1x)$

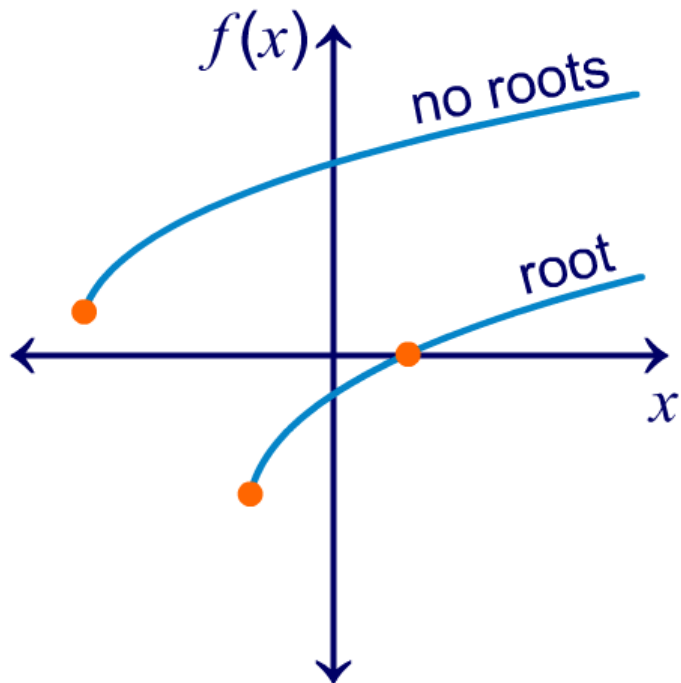
show

show





general square root function: $f(x) = a\sqrt{bx - h} + k$



vertical asymptote: none

horizontal asymptote: none

domain: $[h/b, \infty)$ for $b > 0$
 $(-\infty, h/b]$ for $b < 0$

range: $[k, \infty)$ for $a > 0$
 $(-\infty, k]$ for $a < 0$

graph begins at: $(h/b, k)$

roots: $x = (k^2 + a^2h)/a^2b$ OR
 none



Match functions to graphs

Match each translated radical function to its graph

$$f(x) = \sqrt{x-1}$$

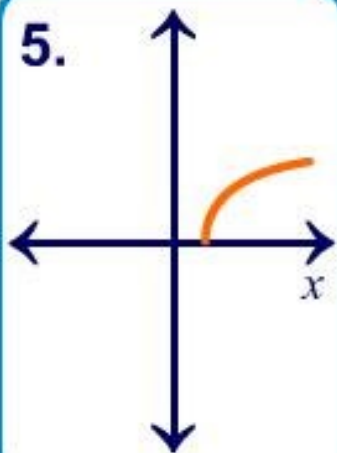
$$g(x) = \sqrt{x+1}$$

$$h(x) = \sqrt{x-1} + 1$$

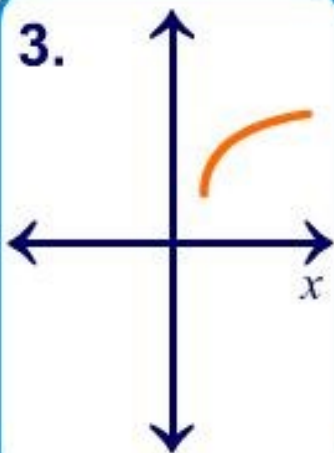
$$k(x) = \sqrt{x+1} - 1$$

$$n(x) = \sqrt{x} - 1$$

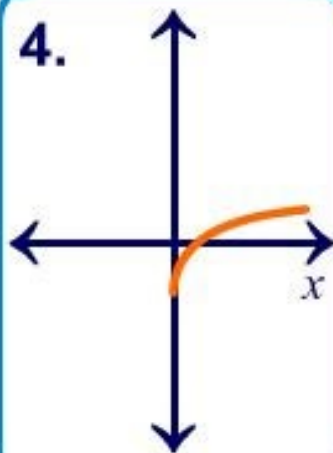
5.



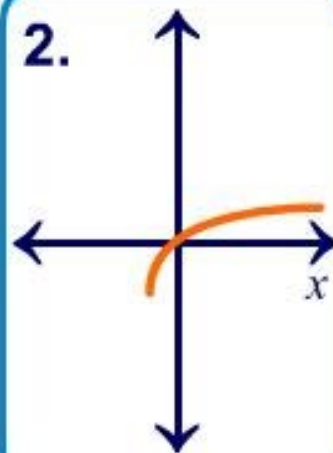
3.



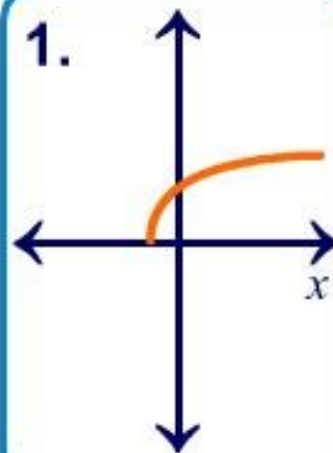
4.



2.



1.

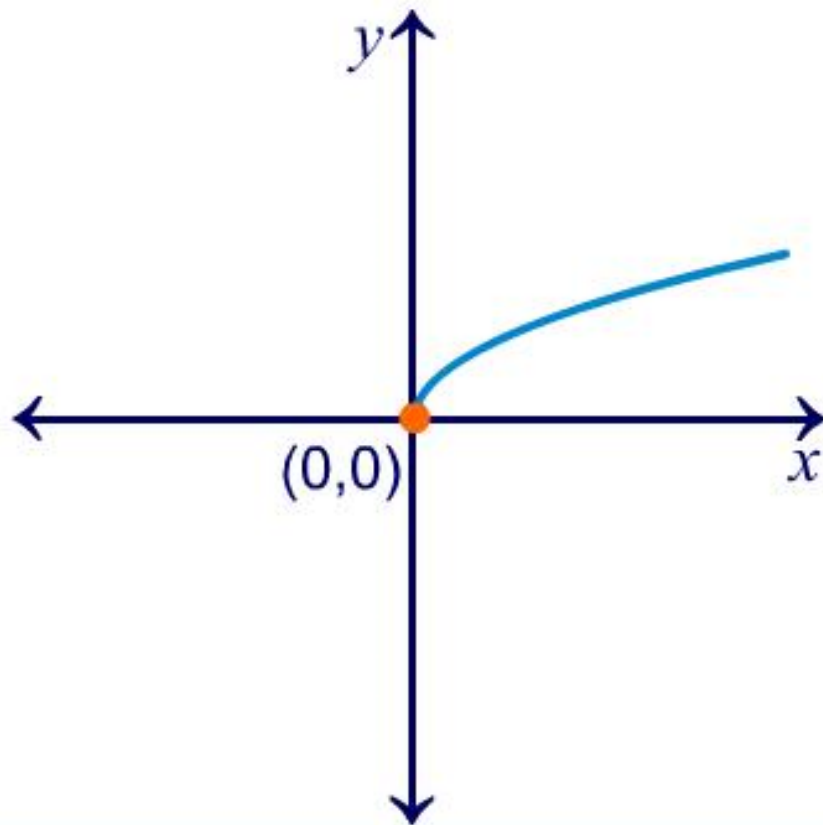


Sketching radical functions

In this activity, try to sketch the graphs of the translated, stretched and reflected radical functions before revealing the answers. Think about which graphs have roots.

Press **start** to begin sketching.

start



Domain and range practice

Drag the correct symbols and numbers into place

	domain	range	
$f(x) = \sqrt{x-3}$	<input type="text"/> <input type="text"/> , <input type="text"/> <input type="text"/>	<input type="text"/> <input type="text"/> , <input type="text"/> <input type="text"/>	[] ()
$g(x) = \sqrt[3]{x-3}$	<input type="text"/> <input type="text"/> , <input type="text"/> <input type="text"/>	<input type="text"/> <input type="text"/> , <input type="text"/> <input type="text"/>	3
$r(x) = \sqrt{3x+3}$	<input type="text"/> <input type="text"/> , <input type="text"/> <input type="text"/>	<input type="text"/> <input type="text"/> , <input type="text"/> <input type="text"/>	-3
$s(x) = \sqrt[4]{-(x-3)}$	<input type="text"/> <input type="text"/> , <input type="text"/> <input type="text"/>	<input type="text"/> <input type="text"/> , <input type="text"/> <input type="text"/>	0
$t(x) = -3\sqrt{x}+3$	<input type="text"/> <input type="text"/> , <input type="text"/> <input type="text"/>	<input type="text"/> <input type="text"/> , <input type="text"/> <input type="text"/>	∞ $-\infty$

