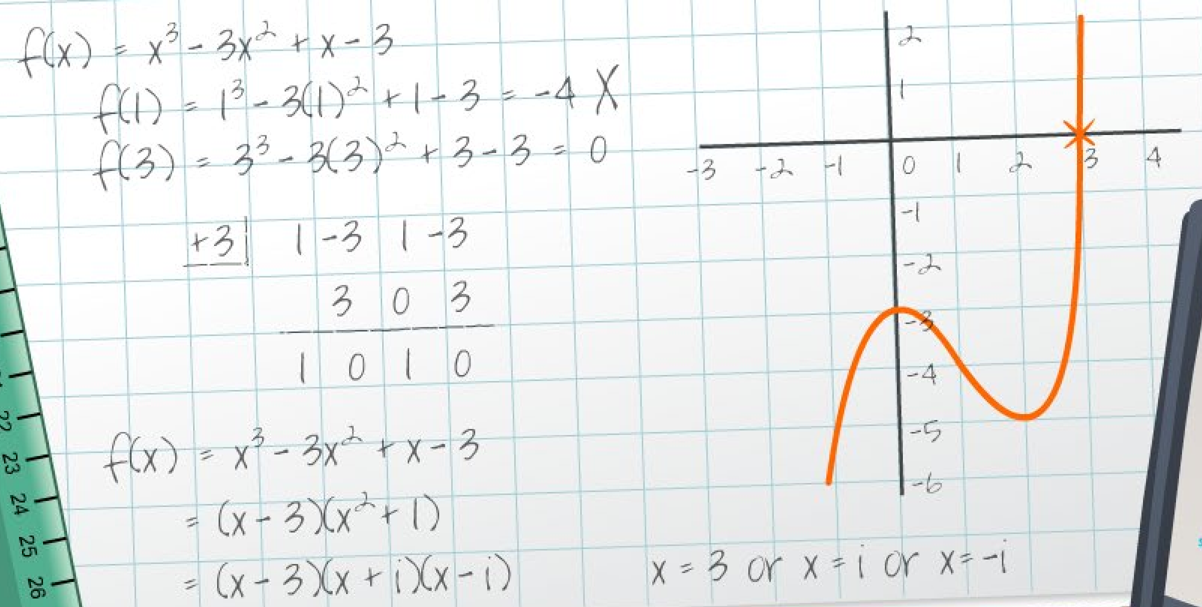
Reciprocal Functions



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

MODELING

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Inverse variation

An airtight 1 dL (deciliter) container with a piston is sealed off, trapping air inside. As the piston is moved, the volume of the container changes. The table shows what happens to the pressure of the trapped air.

V	p	$V \cdot p$
1	1	1
0.5	2	1
0.25	4	1
2	0.5	1
4	0.25	1

What can you say about the product of V and p? Write the function that relates them.

$Vp = 1 \quad \Rightarrow \quad p = 1/V$

Pressure varies **inversely** with volume. In inverse variation, the variables have a constant product.

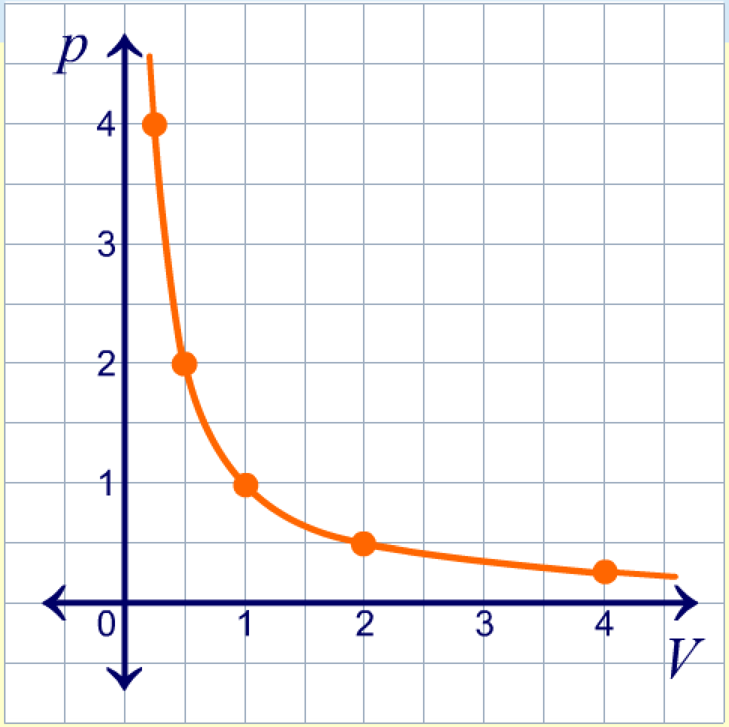
Reciprocal functions

Inverse variation is described by **reciprocal functions**.

Plot the data shown in the table and sketch a reciprocal curve to fit the data.

V	p	$V \cdot p$
1	1	1
0.5	2	1
0.25	4	1
2	0.5	1
4	0.25	1

$p = 1/V$



What is the domain for the sketched curve?

Direct or inverse variation?

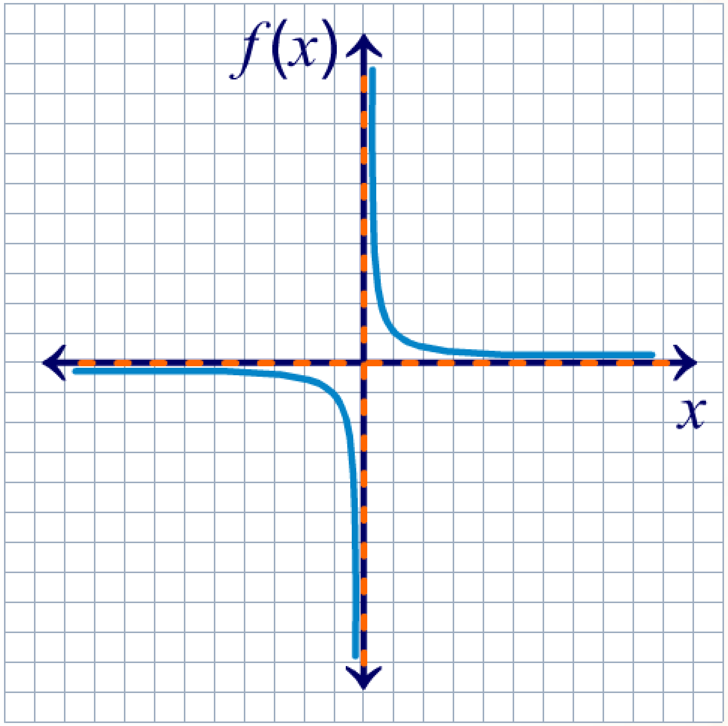
Sort the functions depending on how they vary in x

varies directly in x	varies inversely in x

$n(x) = xy$

S C ?

parent reciprocal function: $f(x) = \dfrac{1}{x}$



vertical asymptote: $x = 0$

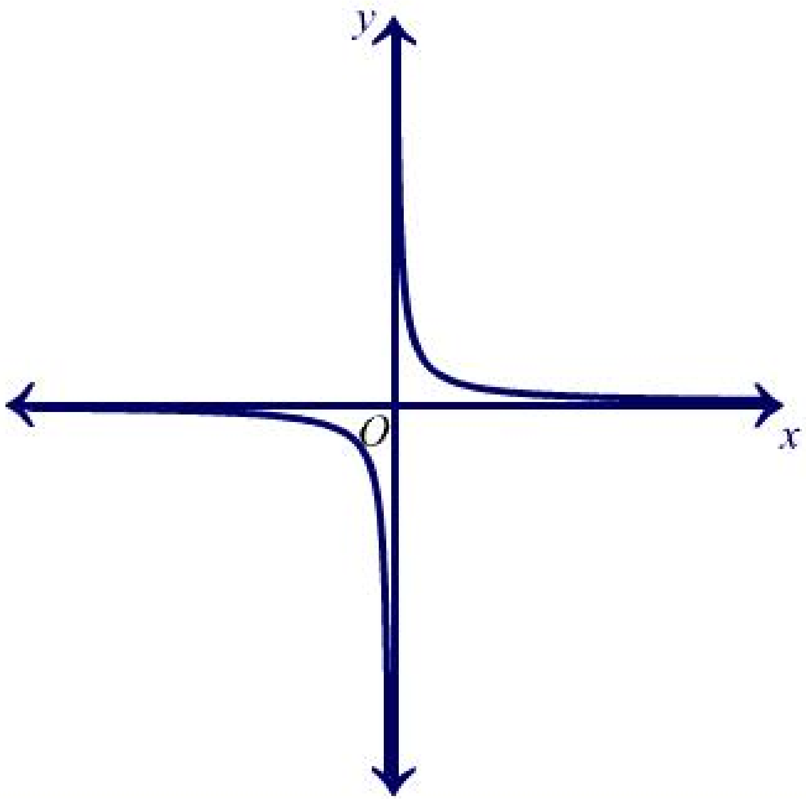
horizontal asymptote: $f(x) = 0$

domain: $(-\infty, 0) \cup (0, \infty)$

range: $(-\infty, 0) \cup (0, \infty)$

roots: none

Stretching reciprocal functions



— $f(x) = \frac{1}{x}$

Press the arrows to transform the function. Press ***show*** *or* ***hide*** *to show or hide the transformed functions.*

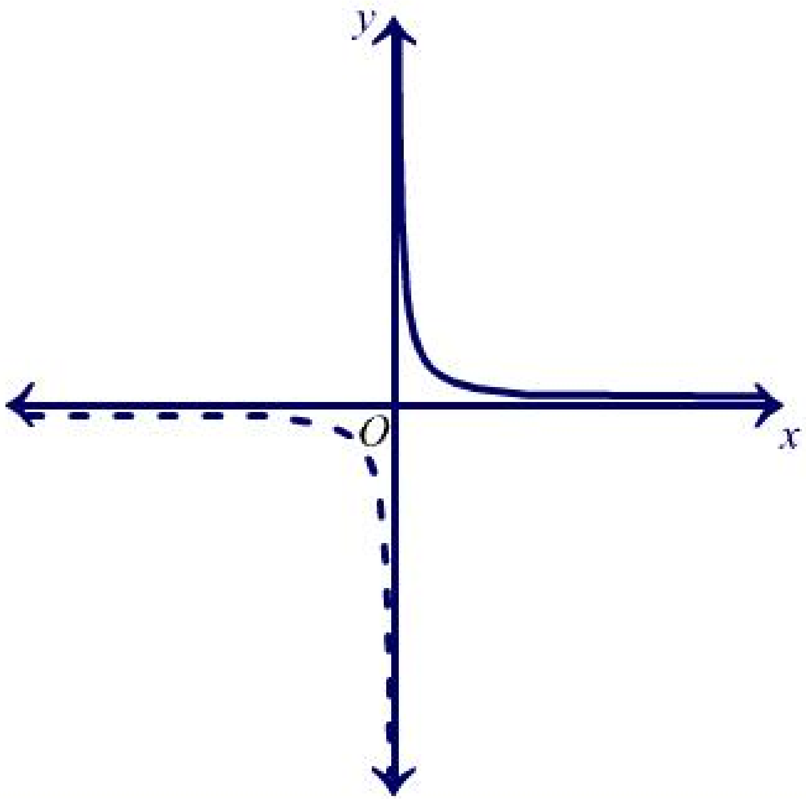
— $y = 1f(x)$

— $y = f(1x)$

show

show

Reflecting reciprocal functions



Press the arrows to change the coefficient.

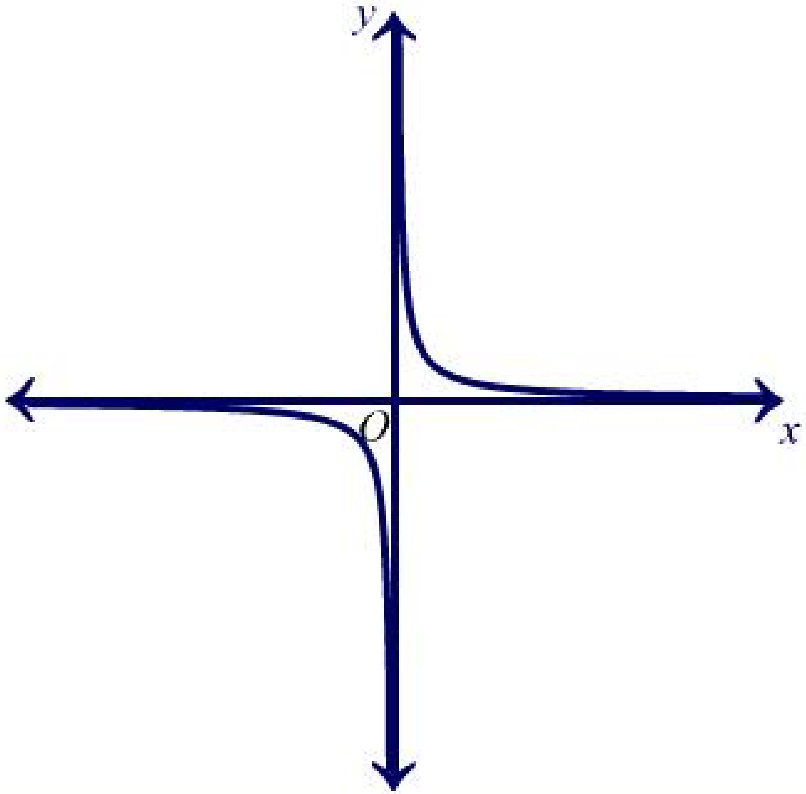
— $f(x) = \frac{1}{x}$

Press ***show*** *or* ***hide*** *to show or hide the transformed functions.*

— $y = f(-x)$ show

— $y = -f(x)$ show

Translating reciprocal functions



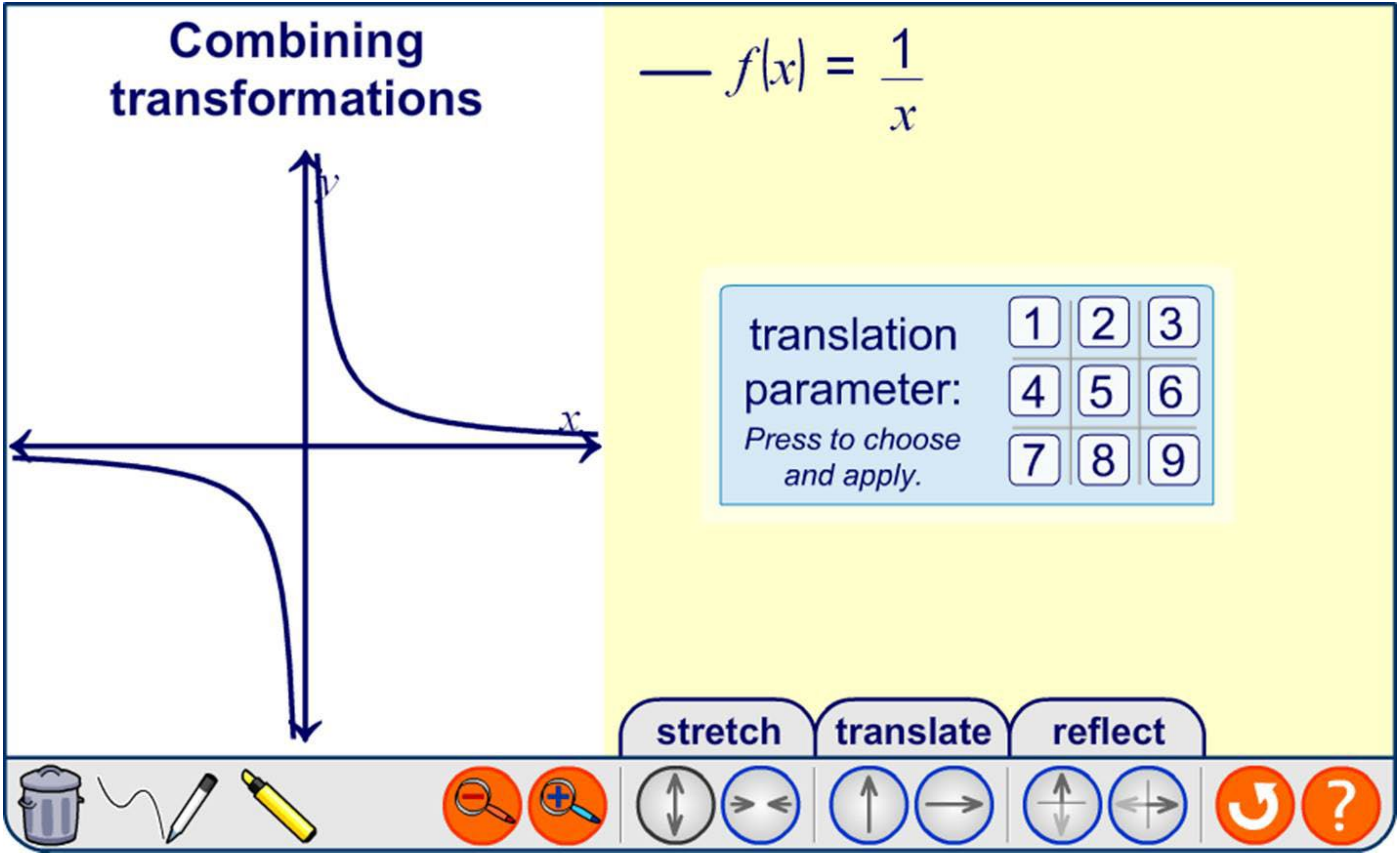
*Press the arrows to transform the function. Press **show** or **hide** to show or hide the transformed functions.*

$f(x) = \frac{1}{x}$

$y = f(x)$ 0 $y = f(x$ 0 $)$

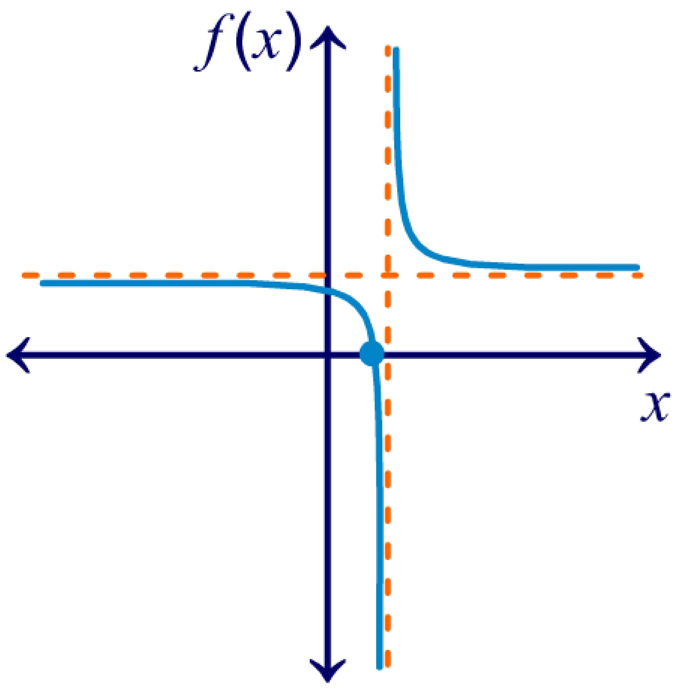
show   show

Combining transformations



General form

general reciprocal function: $f(x) = \dfrac{a}{x - h} + k$



vertical asymptote: $x = h$

horizontal asymptote: $f(x) = k$

domain: $(-\infty, h) \cup (h, \infty)$

range: $(-\infty, k) \cup (k, \infty)$

roots: $x = -\dfrac{a}{k} + h$

Finding vertical and horizontal asymptotes

A **vertical asymptote** in a reciprocal function occurs where it is undefined – where the denominator is zero.

Show that the vertical asymptote of a general reciprocal function is $x = h$.

$$f(x) = \frac{a}{x-h} + k$$

equate the denominator to zero: $x - h = 0$

solve for x: $x = h$ ✓

To understand why a **horizontal asymptote** occurs, examine the behavior of the function as x becomes very large.

Explain why the horizontal asymptote of a general reciprocal function is $y = k$.

$$f(x) = \frac{a}{x-h} + k$$

x is in the denominator, and as the denominator of a fraction become much larger than the numerator, the fraction tends to zero, but does not affect the constant k.

Asymptotes of reciprocal functions

Match the reciprocal functions to their features

horizontal asymptote: $y = -2$
vertical asymptote: $x = 2$

horizontal asymptote: $y = -2$
vertical asymptote: $x = -2$

horizontal asymptote: $y = 2$
vertical asymptote: $x = 0$

horizontal asymptote: $y = 0$
vertical asymptote: $x = 2$

horizontal asymptote: $y = 2$
vertical asymptote: $x = 2$

$$\frac{2}{x-2}$$

$$\frac{1}{x-2} - 2$$

$$\frac{1}{x-2} + 2$$

$$\frac{1}{x+2} - 2$$

$$\frac{2}{x} + 2$$