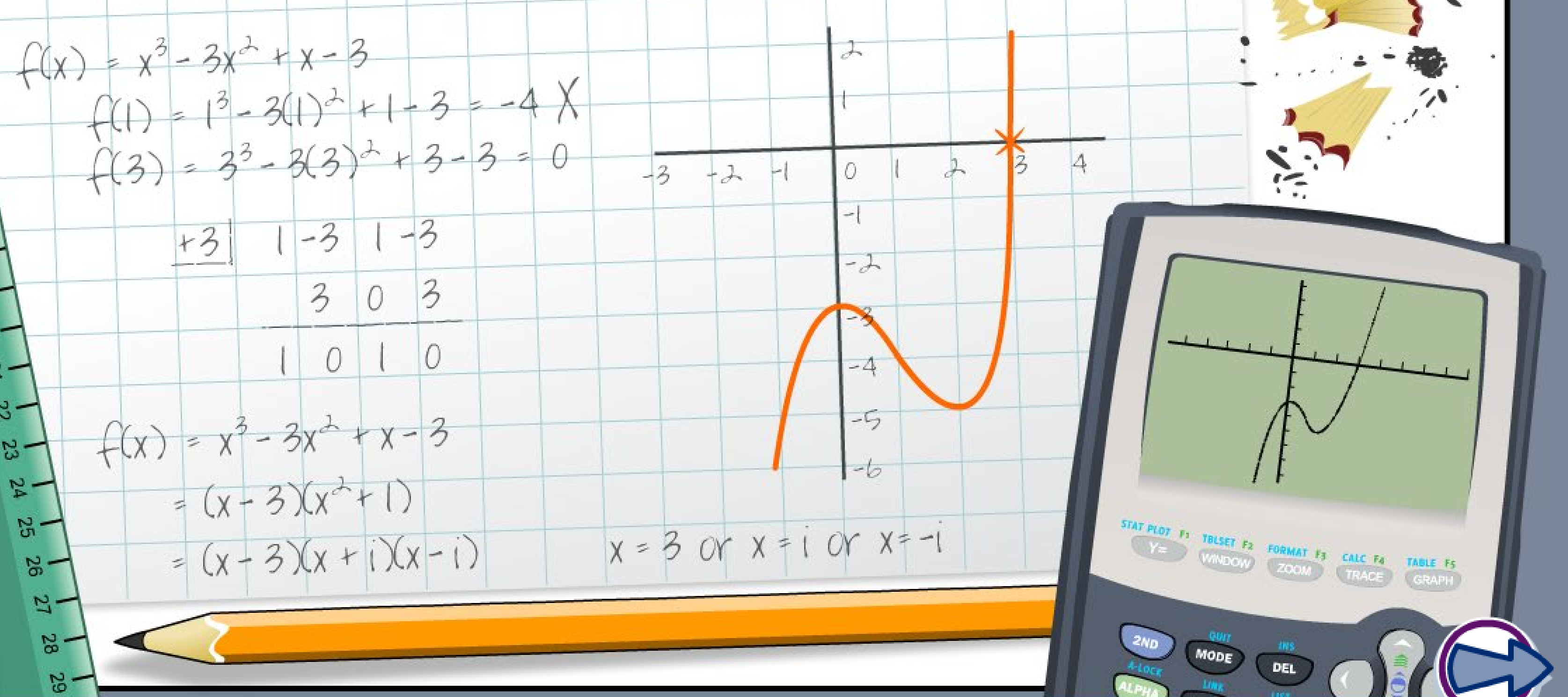
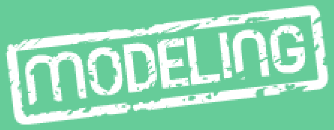
Sketching Graphs



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

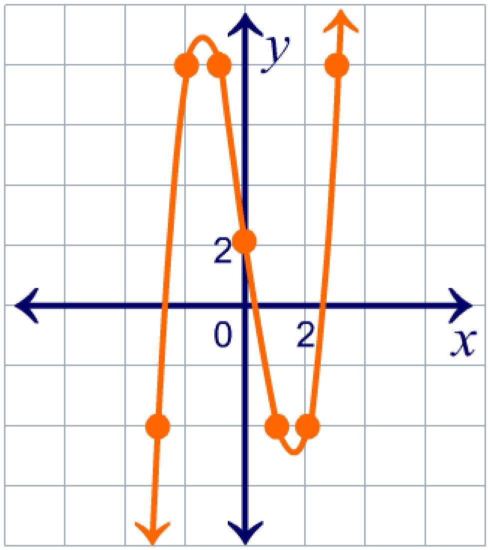
This icon indicates teacher's notes in the Notes field.

How could the graph of $y = x^3 - 7x + 2$ for the interval $-3 \leq x \leq 3$ be plotted?

By creating a table of points, plotting them and joining them with a smooth curve.

x	–3	–2	–1	0	1	2	3
$y = x^3 - 7x + 2$	–4	8	8	2	–4	–4	8

These values of x and y correspond to the coordinates of points that lie on the curve.

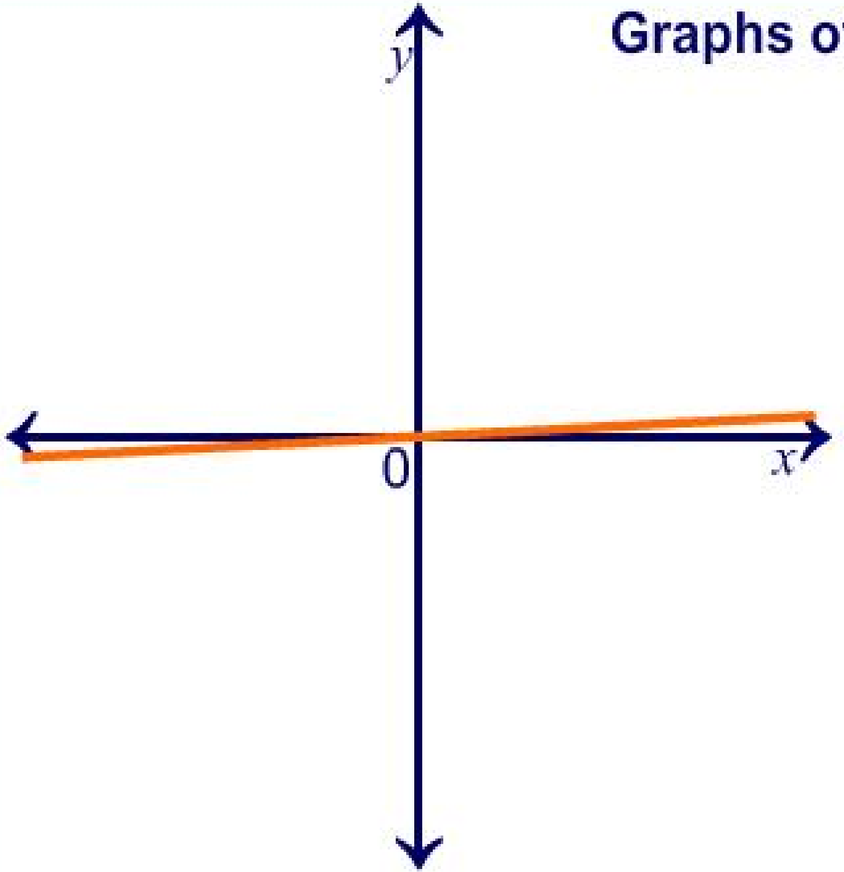


Sketching polynomial functions

When the general shape of a graph is known it is more usual to **sketch** the graph using key features.

- **The x- and y-intercepts.**
  Substitute $y = 0$ or $x = 0$ into the equation.
- **Find the multiplicity of the x-intercepts.**
  A zero with an even multiplicity means that the graph is tangent to the x-axis at that point, a zero with an odd multiplicity is where the graph crosses the x-axis.
- **Determine the end behavior.**
  The trend of y-values as x approaches positive or negative infinity. Look at the coefficient, a, and degree, n, of the polynomial to determine the end behavior.

Graphs of the function $y = kx^n$



$y = 0\ x$

Press x and use the scrollers to change its power, or press the coefficient and use the scrollers to change the coefficient.

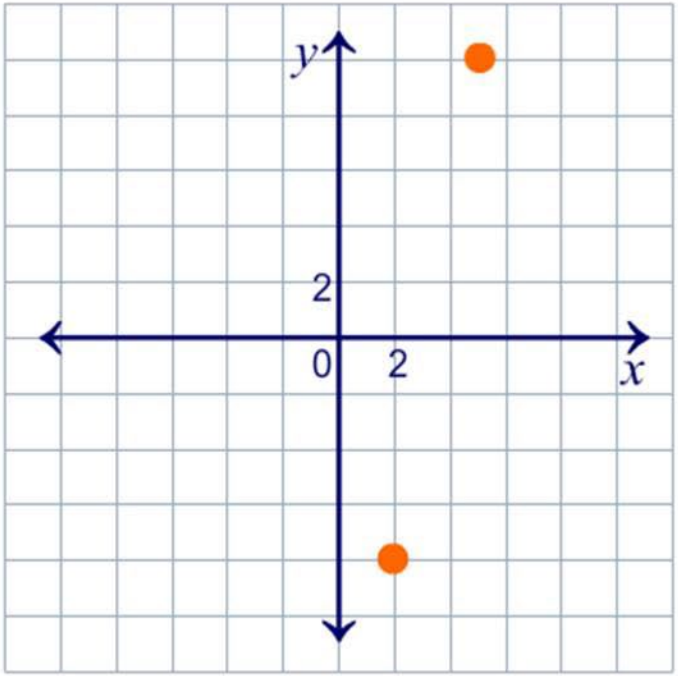
Sketching a parabola using its characteristics

Graph a parabola with a vertex at (2, –8) that passes through (5, 10).

Steps:

1) write in vertex form
2) write in standard form
3) find the y-intercepts
4) find the x-intercepts
5) find the axis of symmetry
6) determine end behavior.

Press the **forward arrow** to move through the steps.



vertex: (2, –8)
passes through: (5, 10)

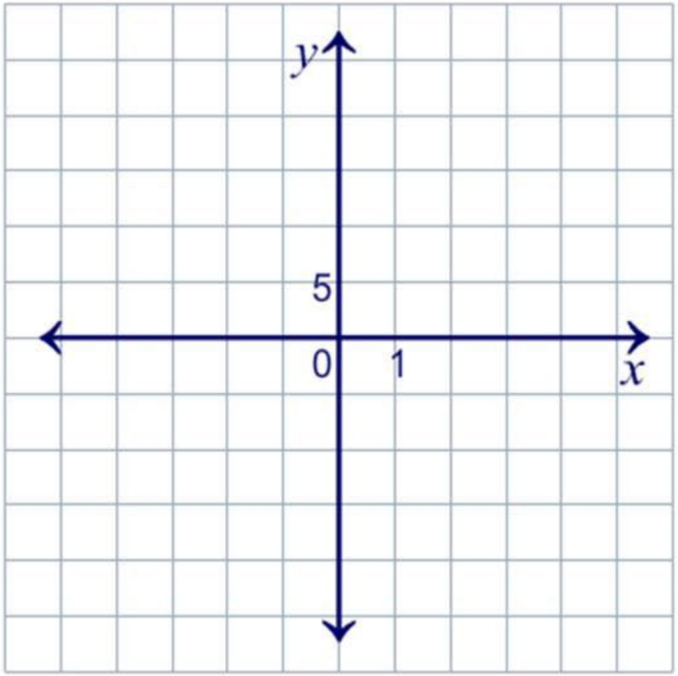
Sketching a parabola using the standard form

Sketch the graph of
$f(x) = -2x^2 - 12x - 16$.

Steps:

1) find the vertex
2) find the axis of symmetry
3) find the y-intercepts
4) find the x-intercepts
5) determine end behavior.

Press the **forward arrow** to move through the steps.



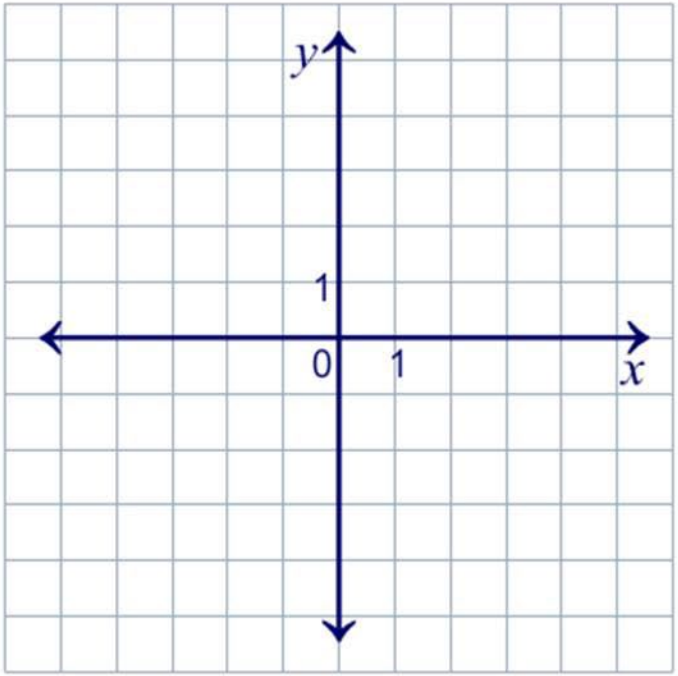
$f(x) = -2x^2 - 12x - 16$

Sketching a cubic graph

Sketch the graph of
$f(x) = x^3 + 2x^2 - 3x$.

Steps:
1) find the y-intercepts
2) find the x-intercepts
3) determine end behavior.

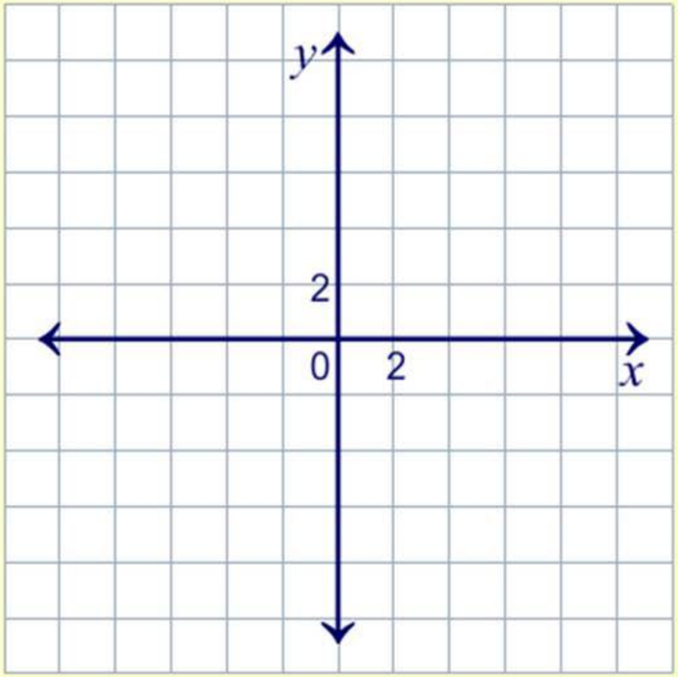
Press the **forward arrow** to move through the steps.



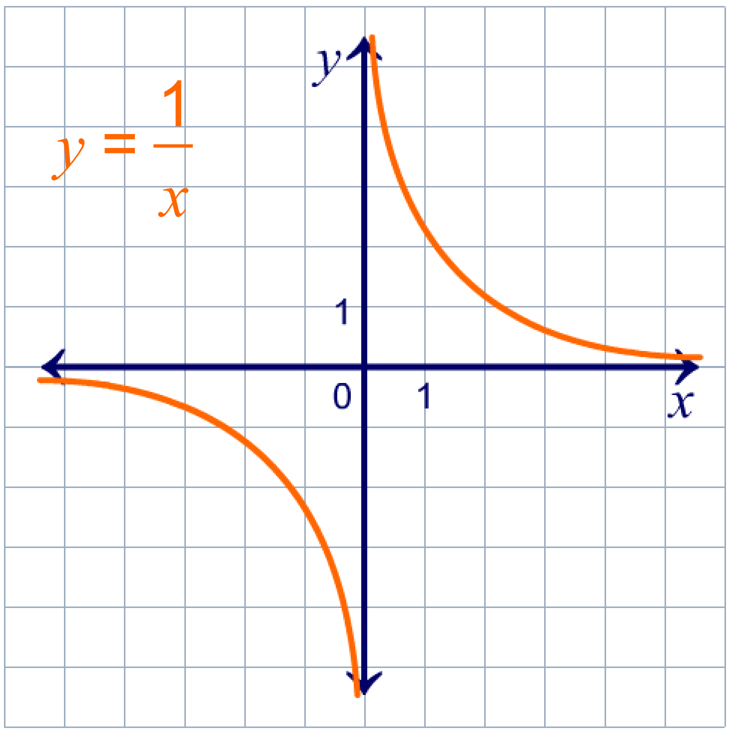
$f(x) = x^3 + 2x^2 - 3x$

Sketching a quartic equation

Sketch the graph of $y = {}^1/_9(x + 3)^2(x - 3)^2$.



Describe the key features of the graph of $y = {}^1/_x$.



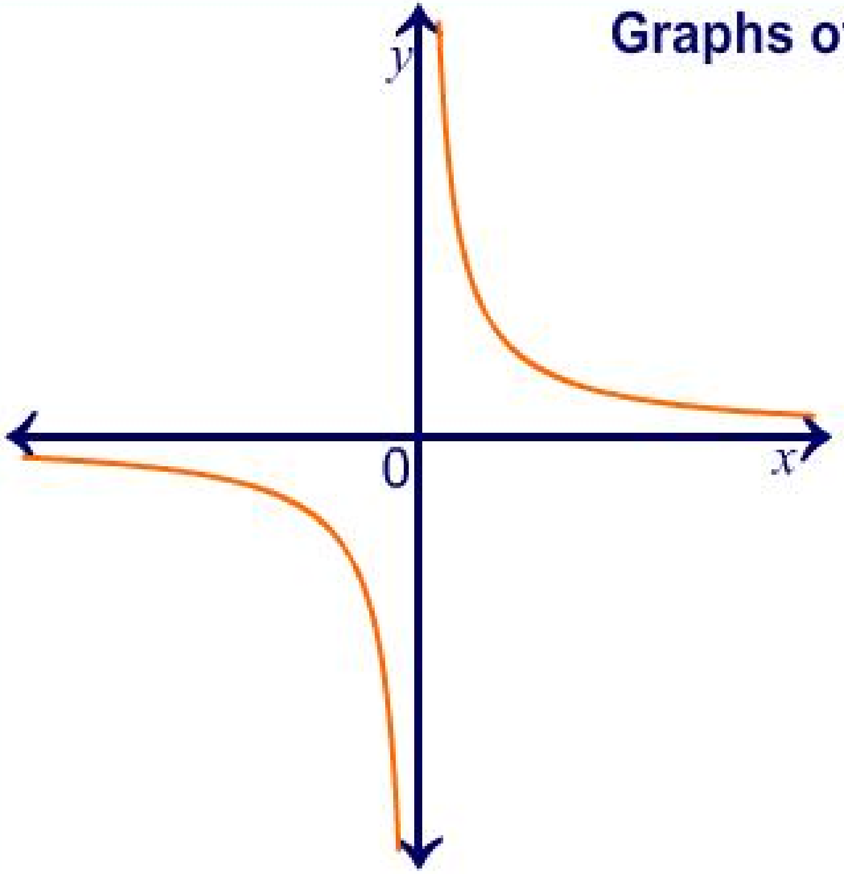
The curve gets closer and closer to the x- and y-axes but never touches them.

The x- and y-axes form the vertical and horizontal **asymptotes**.

The graph of $y = {}^1/_x$ is an example of a **discontinuous** function.

Describe what discontinuous means.

Graphs of the function $y = kx^{-n}$



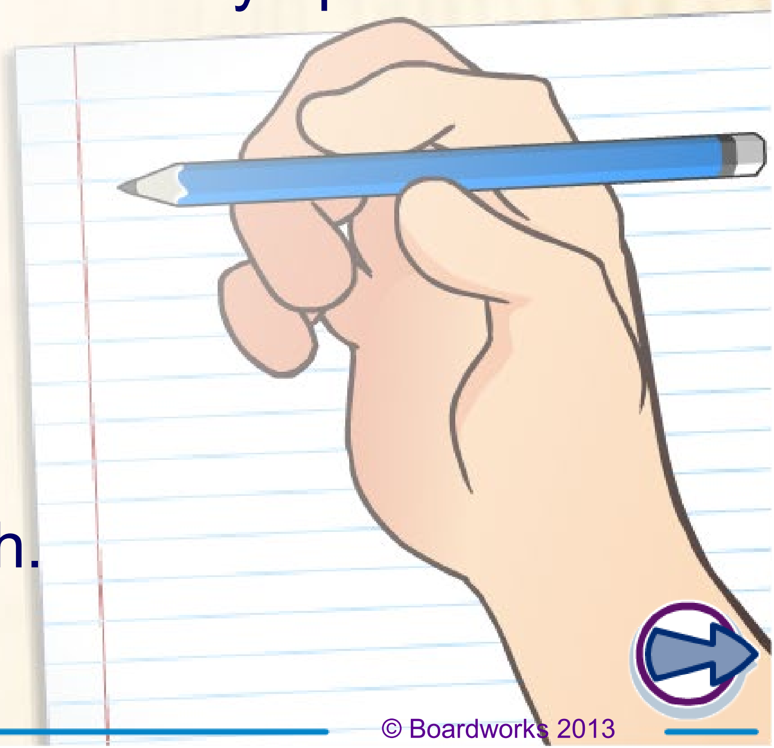
$$y = \frac{1}{x}$$

Press the numerator or denominator and use the scroller to change the value of the numerator and power of x.

Sketching rational graphs

A rational function can be sketched by finding the key characteristics:

1. Use the standard form to find the horizontal asymptotes, which indicates the end behavior of the graph.
2. Use the factored form to find the vertical asymptotes.
3. Find any holes.
4. Find the x- and y-intercepts.
5. Perform a **sign test** on either side of each vertical asymptote to determine the direction of the graph.



Review: holes and asymptotes

Press on a yellow button to review how to find the holes and asymptotes of rational functions.

holes

vertical asymptotes

horizontal asymptotes

Finding asymptotes and holes

Match the rational expression to its feature

$f(x) = \dfrac{(x+9)(x+3)}{(x-3)}$

$f(x) = \dfrac{x^2}{x^2+1}$

$f(x) = \dfrac{(x+1)(x-4)}{(x-4)}$

$f(x) = \dfrac{(x-5)^2}{x^4+5}$

asymptote $x = 3$

asymptote $y = 1$

asymptote $y = 0$

hole $x = 4$

Sketching rational functions (1)

Sketch a graph for the rational function $f(x) = \dfrac{x^2}{x^2 - 25}$.

1. find the horizontal asymptotes:

The degree of the numerator and denominator is the same, so the asymptote is the ratio of their coefficients. Horizontal asymptote at $y = 1$.

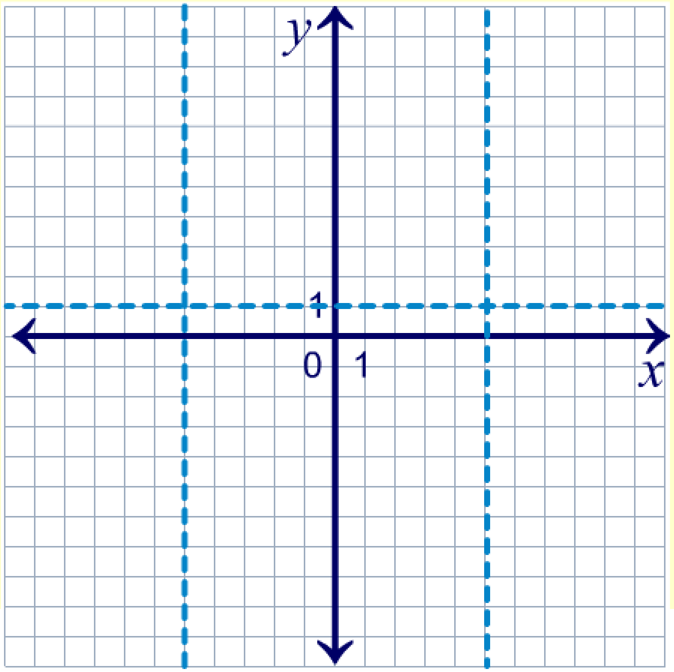
2. factor to find the vertical asymptotes:

$$f(x) = \frac{x^2}{(x+5)(x-5)}$$

Vertical asymptotes at $x = -5$ and $x = 5$.

3. find any holes:

There are no holes.



Sketching rational functions (2)

Sketch a graph for the rational function $f(x) = \dfrac{x^2}{x^2 - 25}$.

4. find the x-intercepts:

$0 = x^2/(x + 5)(x - 5)$

$\boldsymbol{x = 0}$

find the y-intercepts:

$f(0) = (0)^2/(0 + 5)(0 - 5)$

$\boldsymbol{y = 0}$

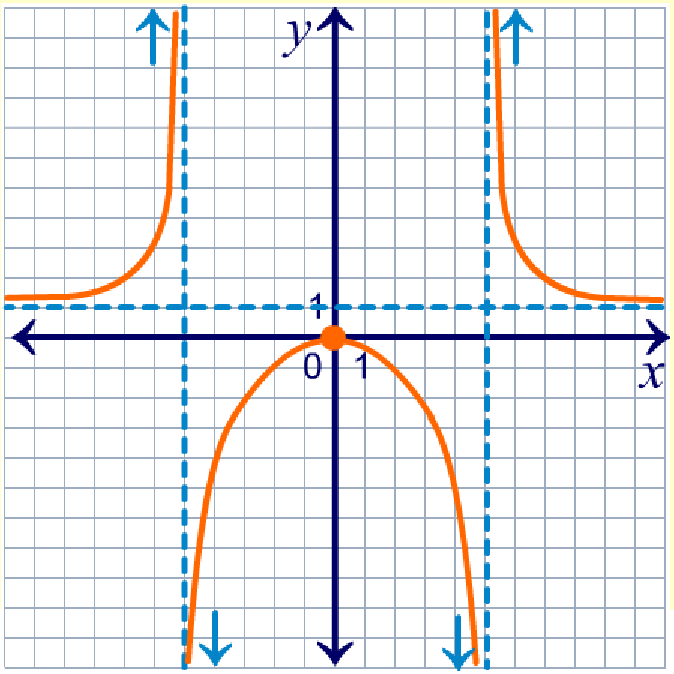
5. perform a sign test on either side of $x = 5$ and $x = -5$:

$$\lim_{x \to -5^-} f(x) = \frac{+}{+} = +\infty$$

$$\lim_{x \to -5^+} f(x) = \frac{+}{-} = -\infty$$

$$\lim_{x \to 5^-} f(x) = \frac{+}{-} = -\infty$$

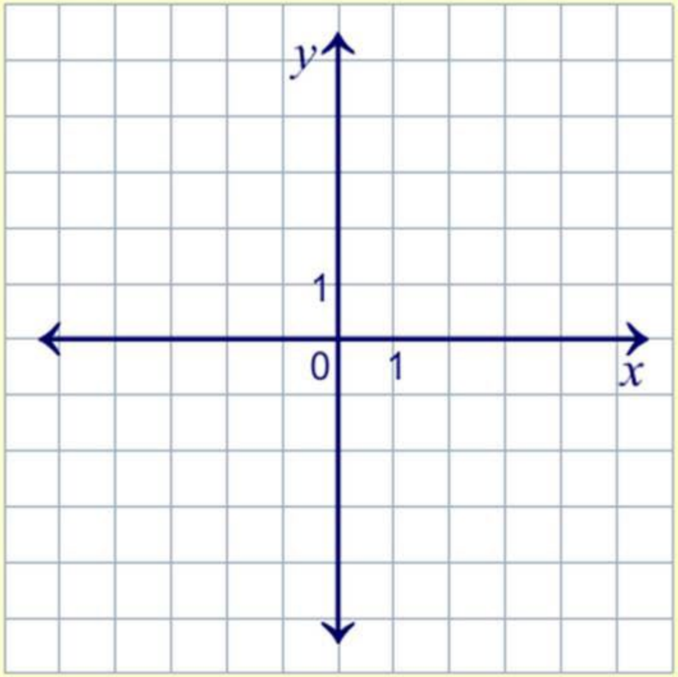
$$\lim_{x \to 5^+} f(x) = \frac{+}{-} = +\infty$$



Sketching a rational function

Sketching a rational function

Sketch the graph of $f(x) = \dfrac{x-1}{(x+2)(x-3)}$



Slant asymptotes

If the degree of the numerator is greater than the degree of the denominator by exactly one, the graph has a **slant asymptote**. It is found using division.

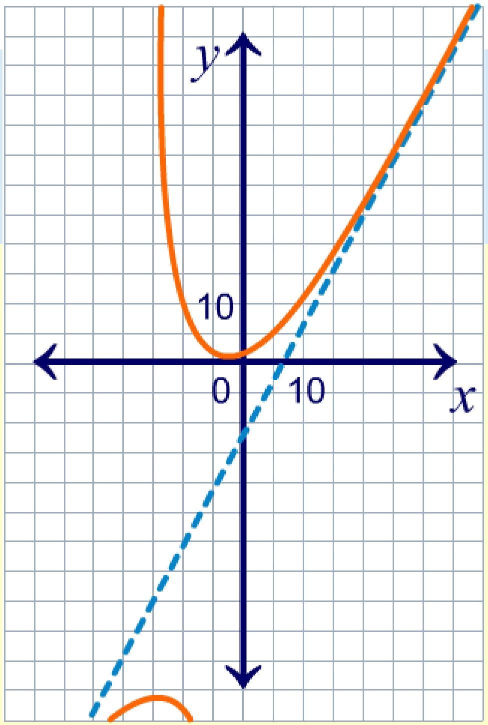
Find the asymptote of $f(x) = \dfrac{3x^2 - 2x + 1}{x + 5}$.

use synthetic division:

–5	3	–2	1
		–15	85
	3	–17	86

this gives: $f(x) = 3x - 17 + \dfrac{86}{x + 5}$

$y = 3x - 17$ is a slant asymptote.



If the degree of the numerator is more than one greater than the denominator, there are no horizontal or slant asymptotes.

Practice sketching rational functions

Press the "W" button next to each radical function to see how to sketch it.

1) $f(x) = \dfrac{x^3}{x^2 - 9}$ W

2) $f(x) = \dfrac{(x - 3)}{x^2 + 2x - 15}$ W

3) $f(x) = \dfrac{3x^6}{x^4 + 10}$ W

Challenge: $f(x) = \begin{cases} \dfrac{1}{x+2}, & x < -2 \\ \sqrt{x+2}, & x \geq -2 \end{cases}$ W

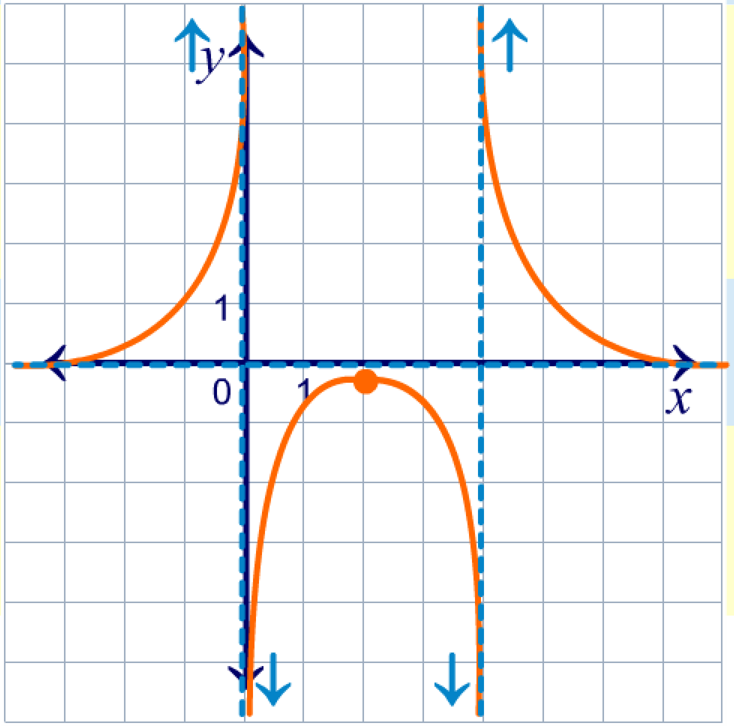
Sketch a rational function with the following features:

- vertical asymptotes at $x = 4$ and $x = 0$
- horizontal asymptote at $y = 0$
- passes through the point $(2, -\frac{1}{4})$.
- as $x \to 0^+$, $f(x) \to -\infty$ and as $x \to 0^-$, $f(x) \to \infty$.
- as $x \to 4^-$, $f(x) \to -\infty$ and as $x \to 4^+$, $f(x) \to \infty$.

Sketch in the asymptotes and given point.

Mark the end behavior using arrows.

Sketch the graph.



What is the equation of this graph?

$$f(x) = \frac{1}{x^2 - 4x}$$