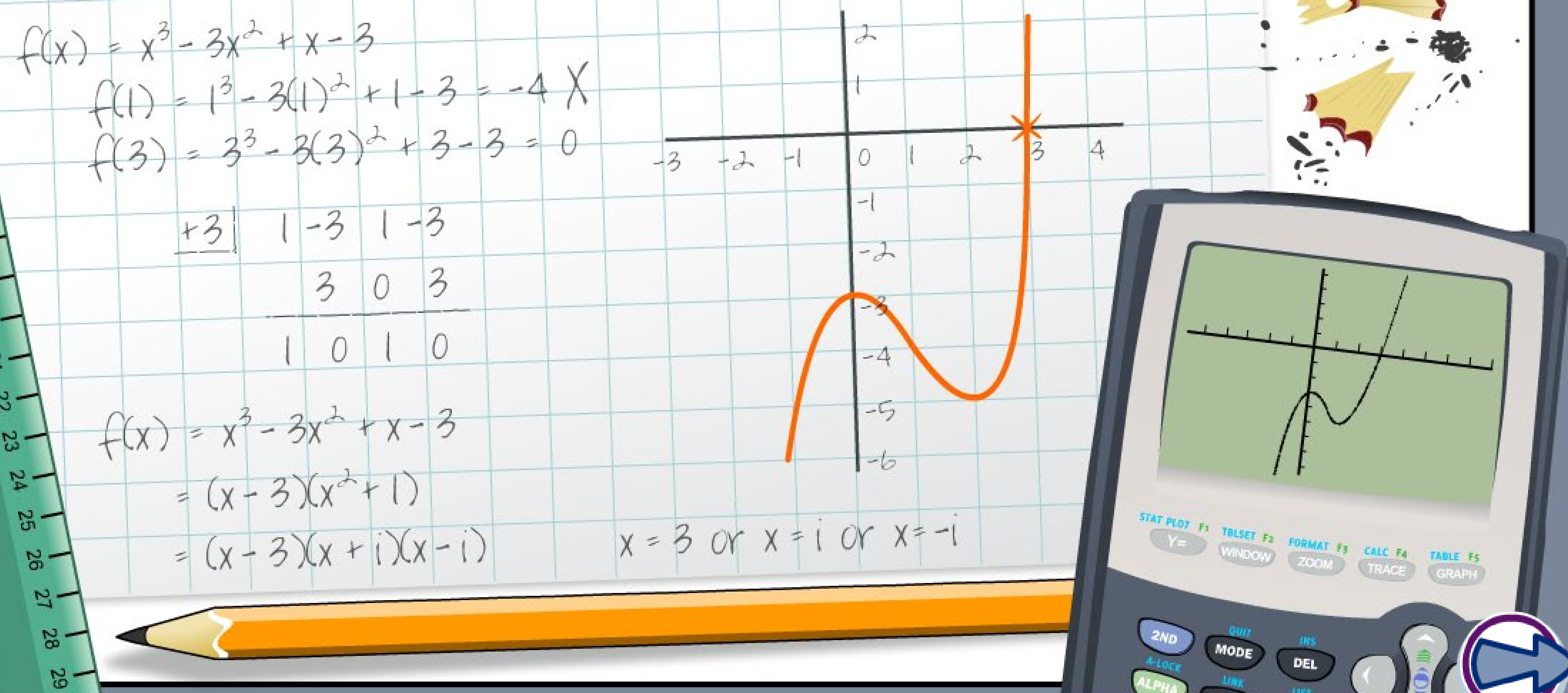
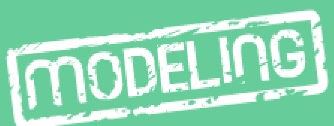
Trigonometric Functions



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

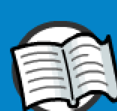
5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

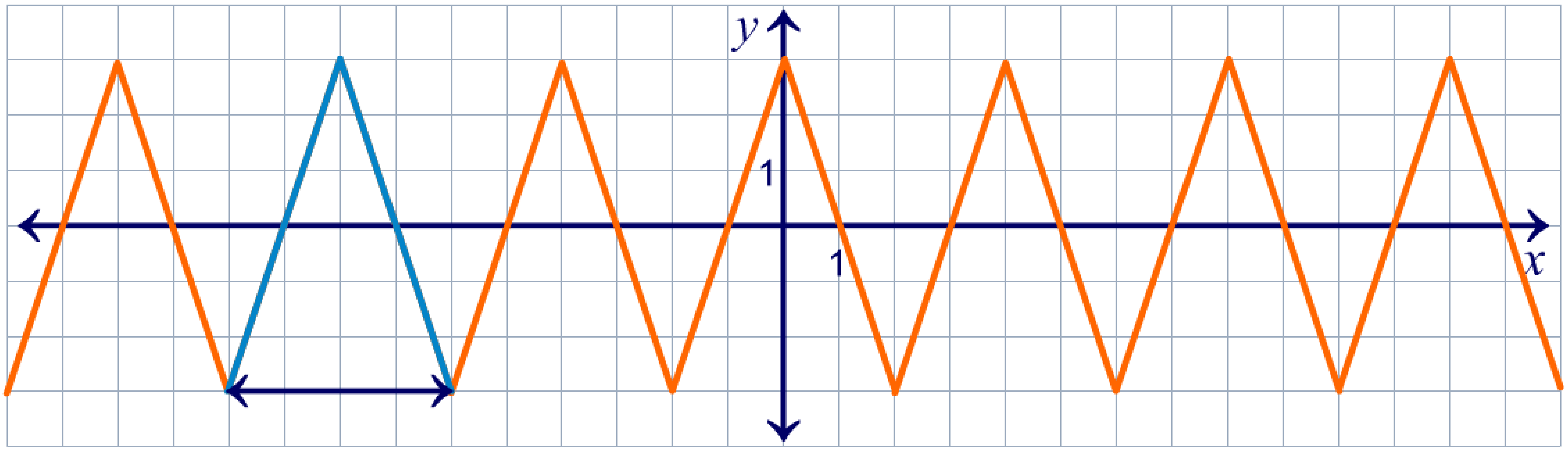
8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

A function with a pattern that repeats at regular intervals is called a **periodic function**.



The length of the interval over which a periodic function repeats is called the **period** of the function.

What is the period of the function shown above?

period = 4

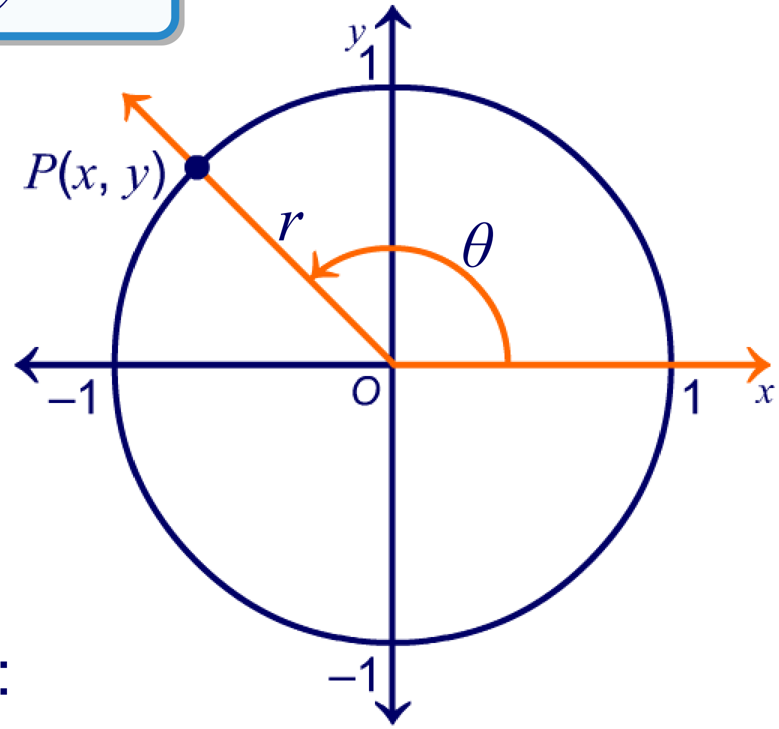
Trigonometric functions of real numbers

The **trigonometric** functions are common periodic functions.

For any **real number θ**, the trigonometric functions are defined:

$\sin\theta = y/r$ $\cos\theta = x/r$ $\tan\theta = y/x$

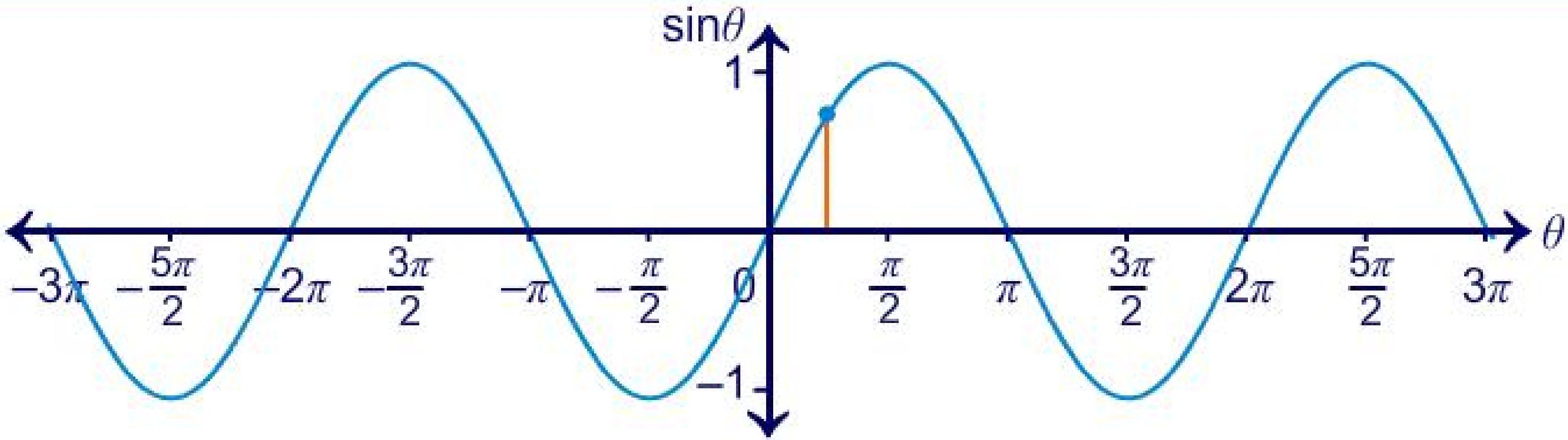
where x and y are the coordinates of any point $P(x, y)$ on the terminal ray of an **angle of θ radians** when it is in standard position, and r is the distance from the origin to P.



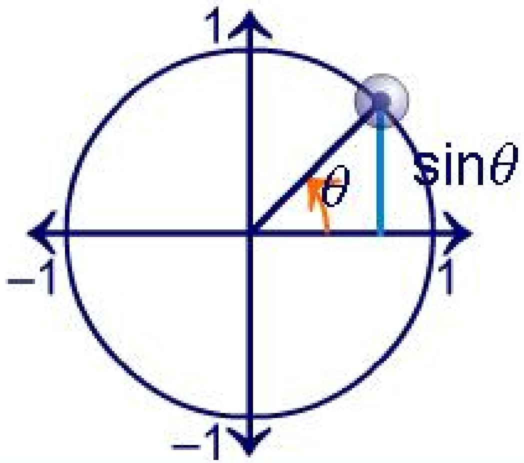
In particular, if P is on the unit circle:

$\sin\theta = y$ $\cos\theta = x$ $\tan\theta = y/x$

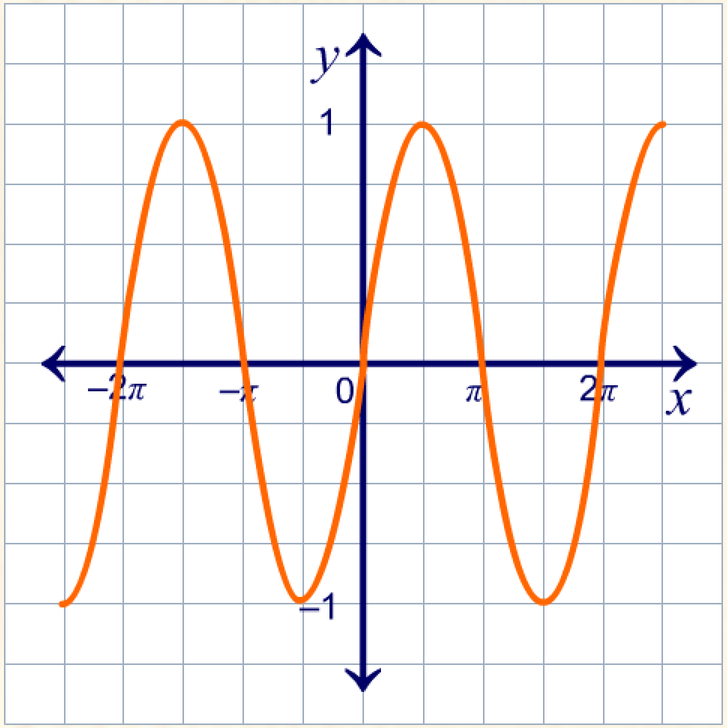
The graph of sinθ



Drag the point around the circle to see how the graph of sine relates to an angle in standard position in the unit circle.



parent function: $f(x) = \sin x$



domain: $(-\infty, \infty)$

range: $[-1, 1]$

asymptotes: none

period: 2π

roots: $x = n\pi$

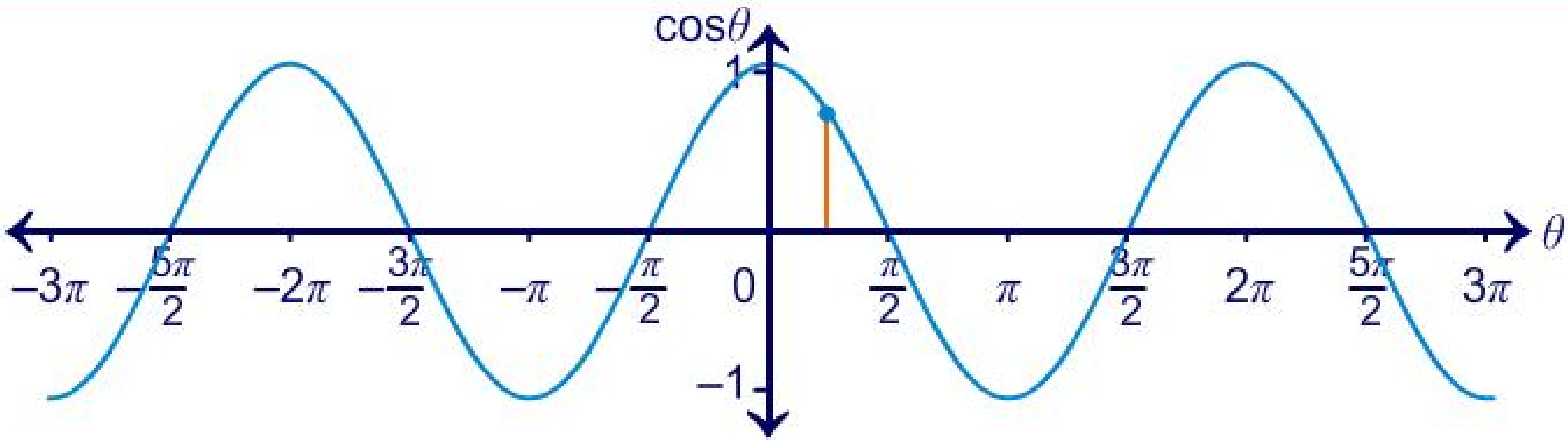
maxima: $x = \pi/2 + 2n\pi$

minima: $x = -\pi/2 + 2n\pi$

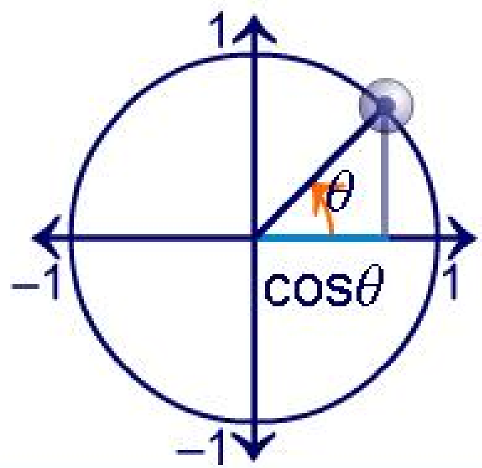
for every integer n

Is $\sin x$ an even or odd function?

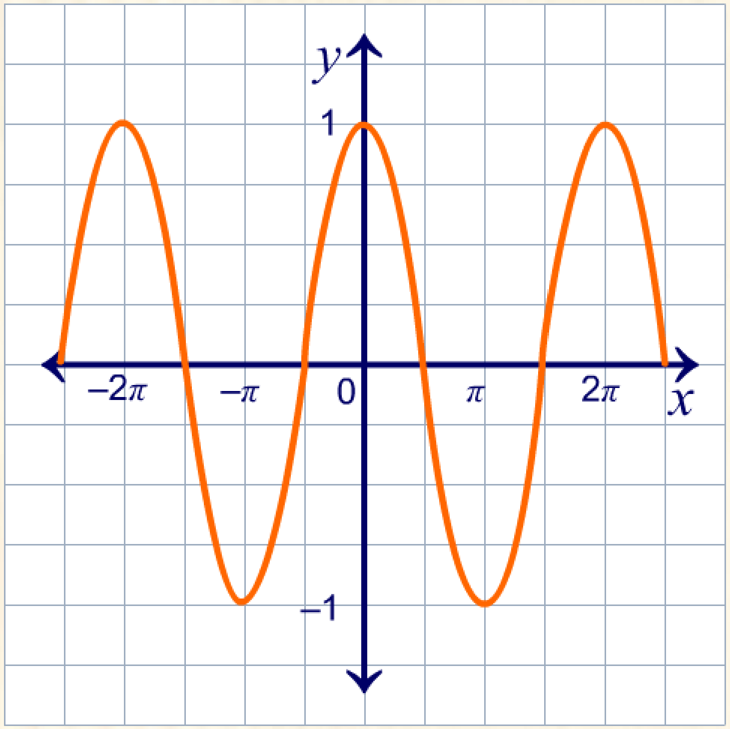
The graph of cosθ



Drag the point around the circle to see how the graph of cosine relates to an angle in standard position in the unit circle.

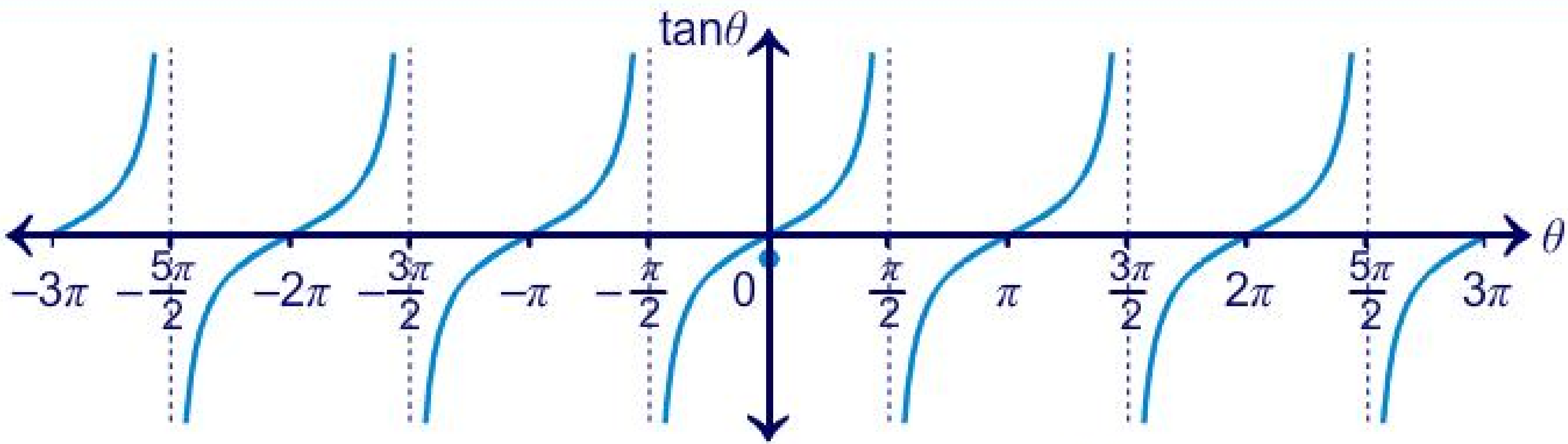


parent function: $f(x) = \cos x$

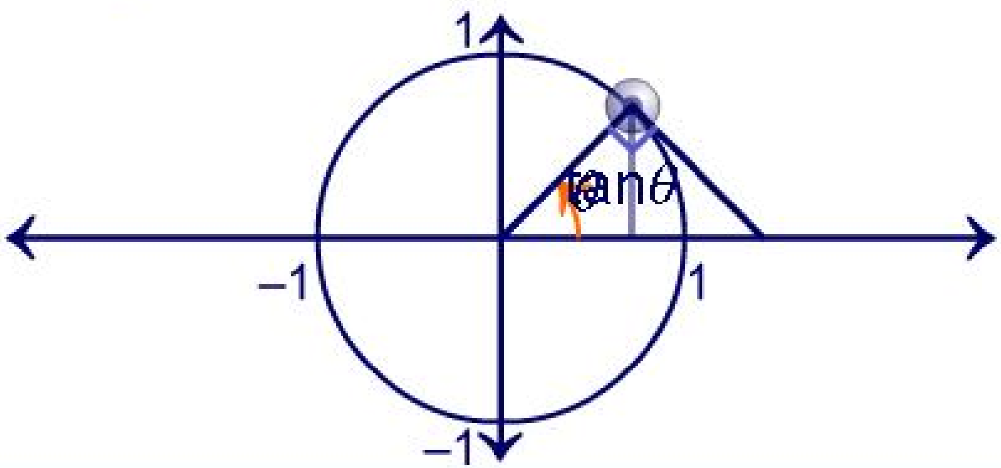


domain:	$(-\infty, \infty)$
range:	$[-1, 1]$
asymptotes:	none
period:	2π
roots:	$x = \pi/2 + n\pi$
maxima:	$x = 2n\pi$
minima:	$x = \pi + 2n\pi$
	for every integer n

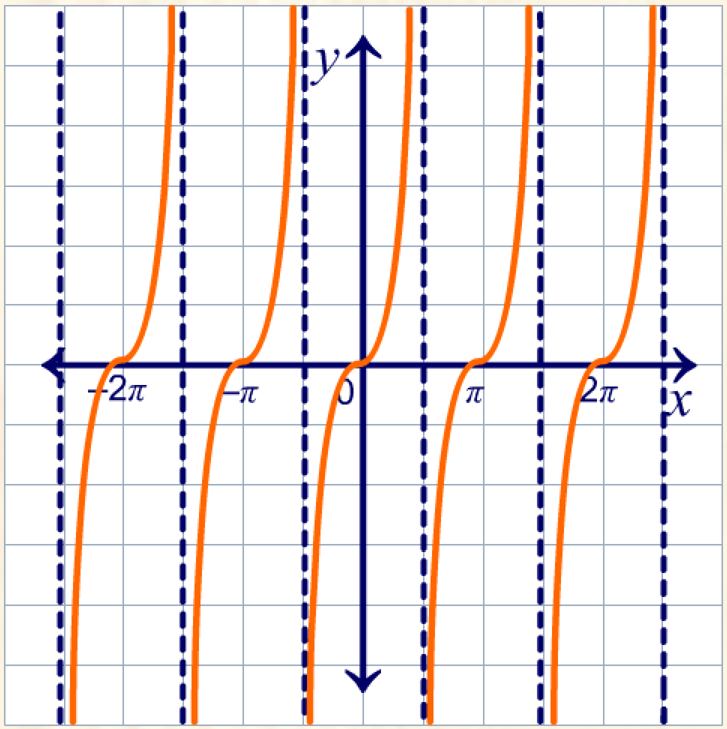
Is $\cos x$ an even or odd function?



Drag the point around the circle to see how the graph of tan relates to an angle in standard position in the unit circle.



parent function: $f(x) = \tan x$



domain: $\left(n\pi - \frac{\pi}{2},\ n\pi + \frac{\pi}{2}\right)$

range: $(-\infty, \infty)$

period: π

roots: $x = n\pi$

horizontal asymptotes: none

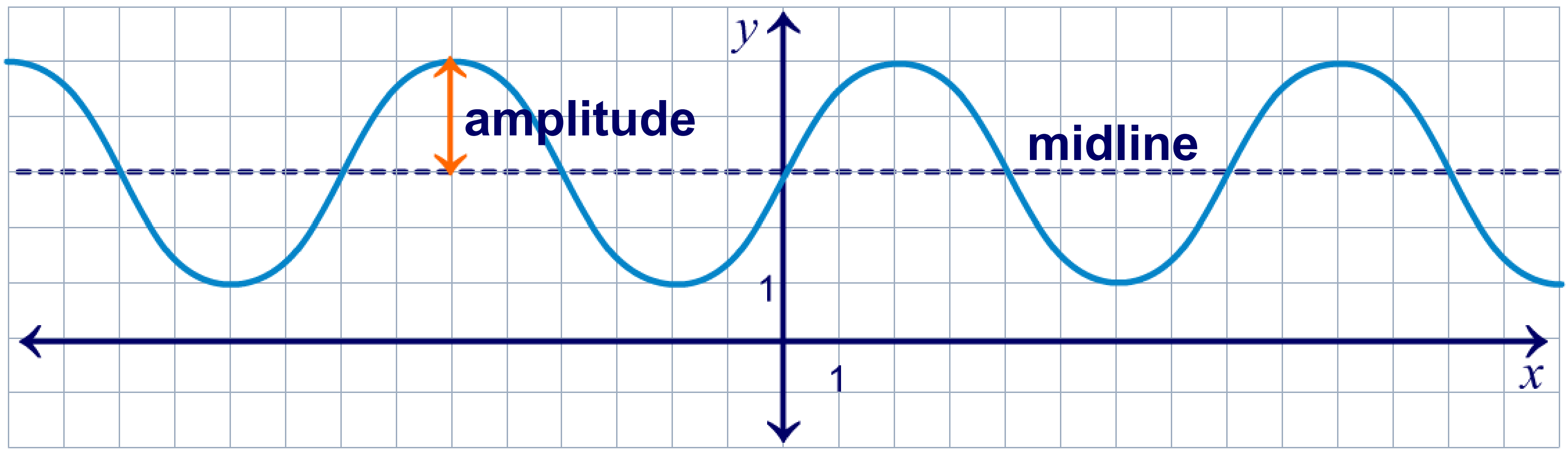
vertical asymptotes: $x = \frac{\pi}{2} + n\pi$

for every integer n

Is tan$x$ an even or odd function?

A key feature of a periodic function is its **amplitude**.

The amplitude of a periodic function is half the difference between the minimum and maximum y values, $(\max_y - \min_y)/2$.



The **midline** of a periodic function is the line at the midpoint of the range. $y = \min_y + (\max_y - \min_y)/2$

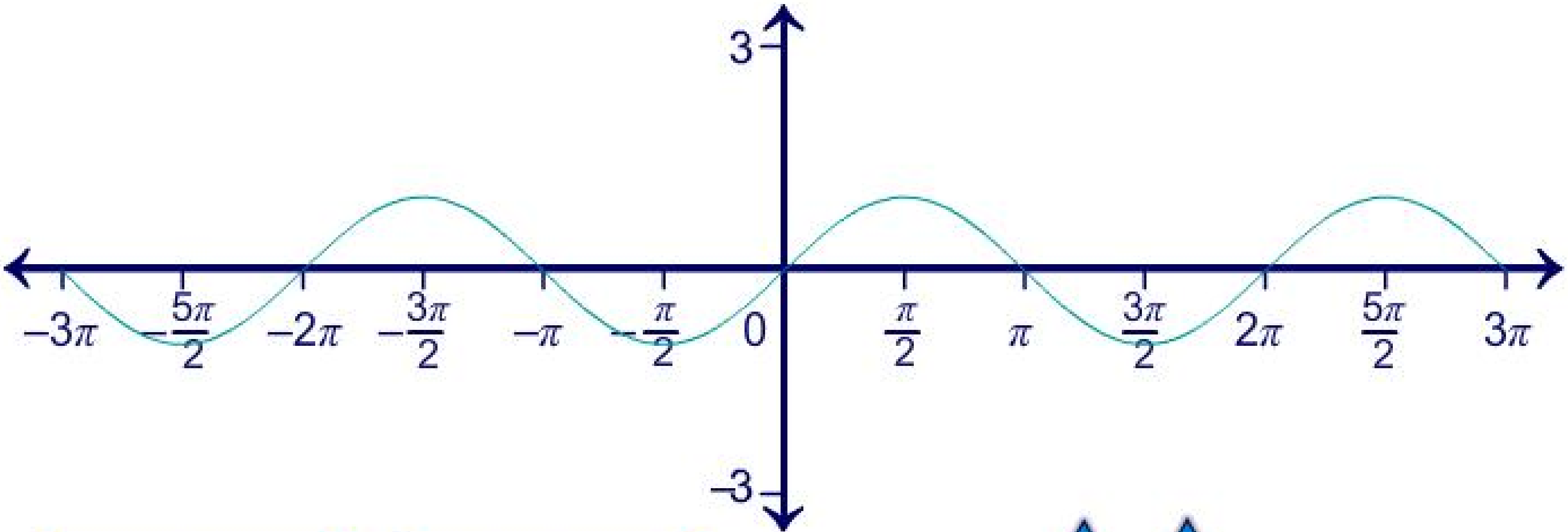
What is the amplitude of the graph shown?
What is the equation of its midline?

Are these claims about the trig functions true or false?

1. sin$x$ and cos$x$ have the same midline. ?
2. sin$x$ and cos$x$ have an amplitude of 2. ?
3. The midline of cos$x$ is y = 1. ?
4. The period of tan$x$ is double the period of sin$x$. ?
5. cos$x$ has a period of 2π. ?

true false

Transforming trigonometric functions



$n \sin x$ | $\sin(nx)$

$\sin x + n$ | $\sin(x + n)$

$f(x) = 1 \sin x$

Which key features of the trigonometric functions does each transformation affect?

$f(x) + n$	amplitude
$nf(x)$	midline
$f(nx)$	period and frequency

Analyzing functions

Graph $f(x) = -3\cos 2x$.

1) Find the domain, range, period and amplitude.

2) Find the x-values where the minimum and maximum values occur in the interval $[0, 2\pi]$.

3) State the zeros in the interval $[0, 2\pi]$.

1) domain: $(-\infty, \infty)$

range: $[-3, 3]$

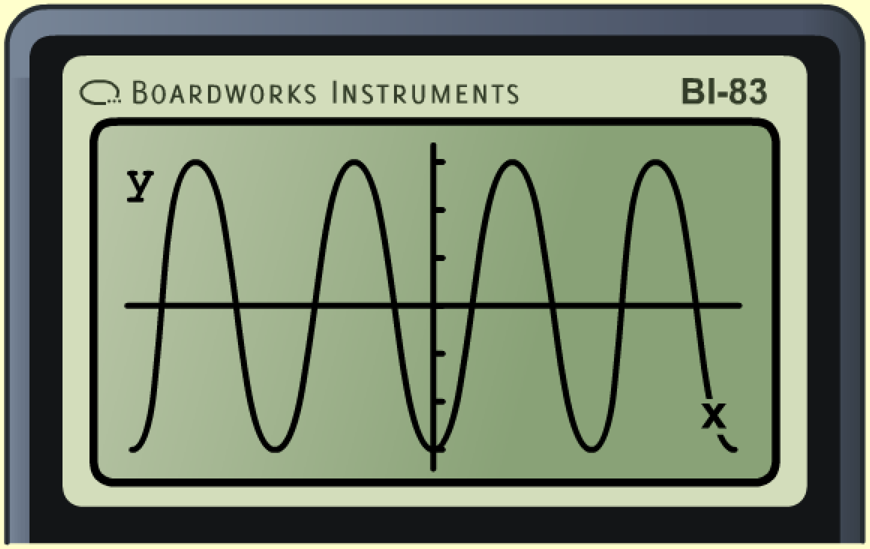
period: π

amplitude: 3

2) minima: $x = 0, \pi, 2\pi$

maxima: $x = \pi/2, 3\pi/2$

3) zeroes: $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$



General form

Sine and cosine functions can be written in the following form:

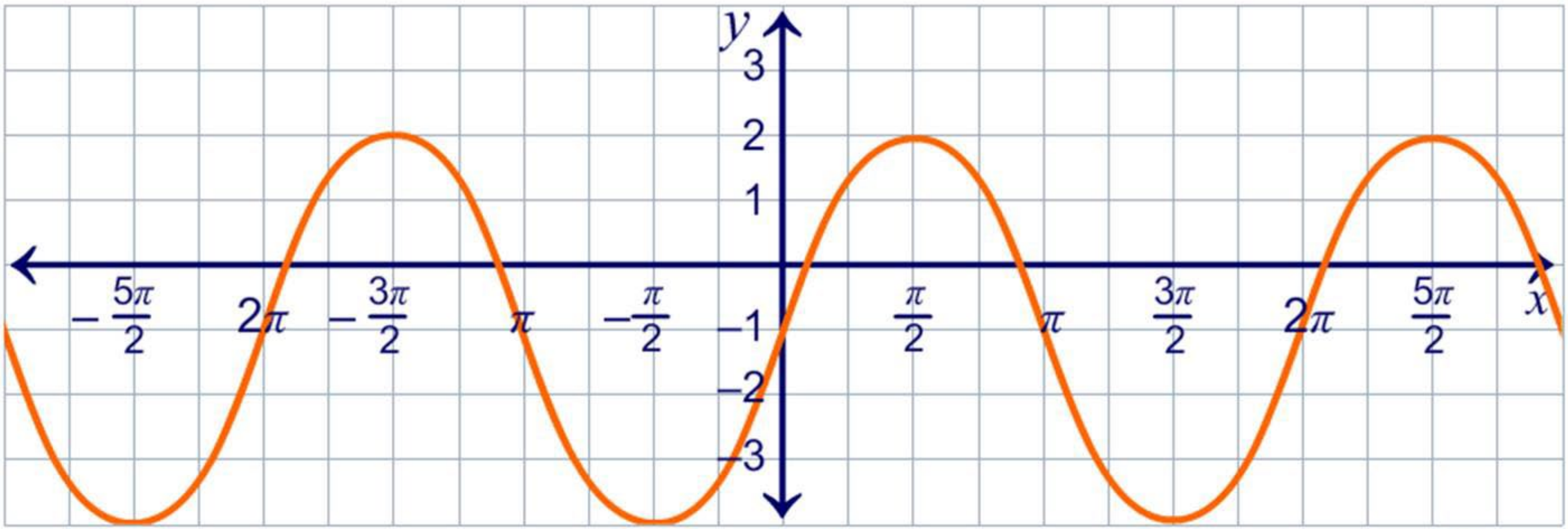
$y = a\sin(b(\theta + c)) + d$ with $a \neq 0$, $b > 0$, and θ in radians

The variables give key features of the graph:

c	period
d	amplitude
b	number of cycles between 0 and 2π
$\lvert a \rvert$	vertical shift
$2\pi/b$	horizontal shift

Match the graph to the equation

Question: 1/6 **Which equation matches the graph?**



A) $y = \sin(3x) + 1$

B) $y = 3\sin x - 1$

C) $y = 3\cos(x) - 1$

D) $y = \cos(3x) + 1$