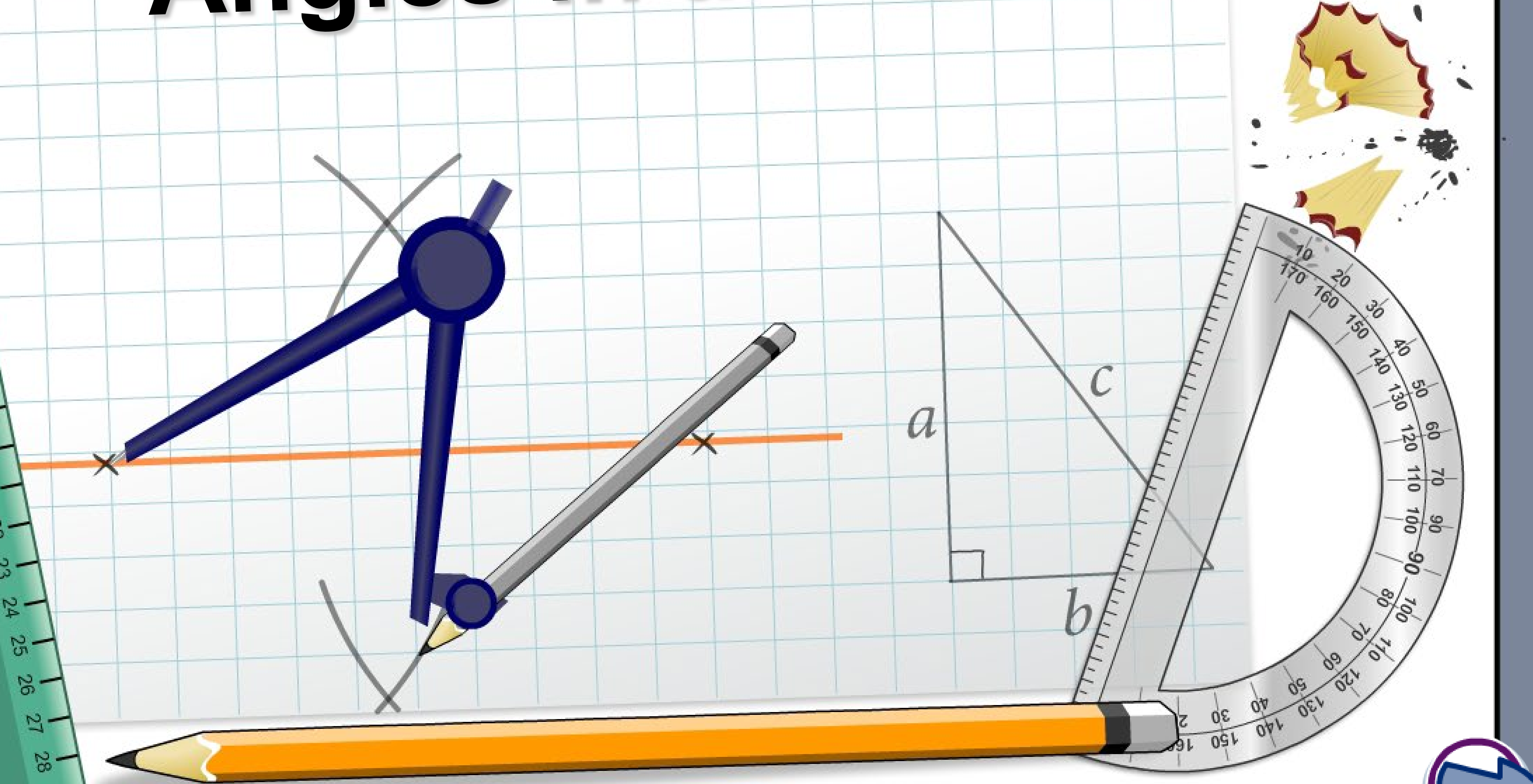
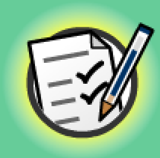
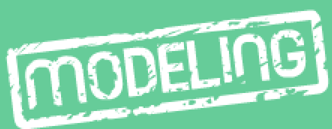
Angles in a Circle



Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

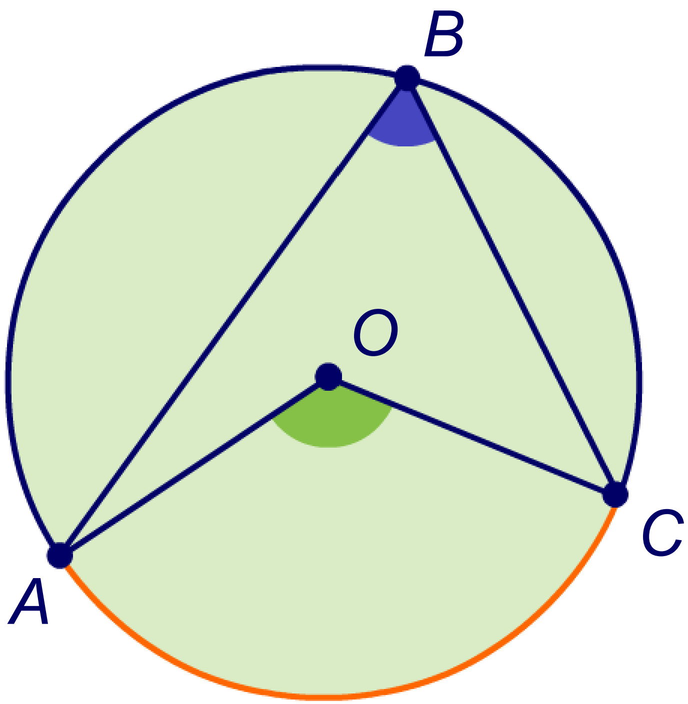
This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Definitions

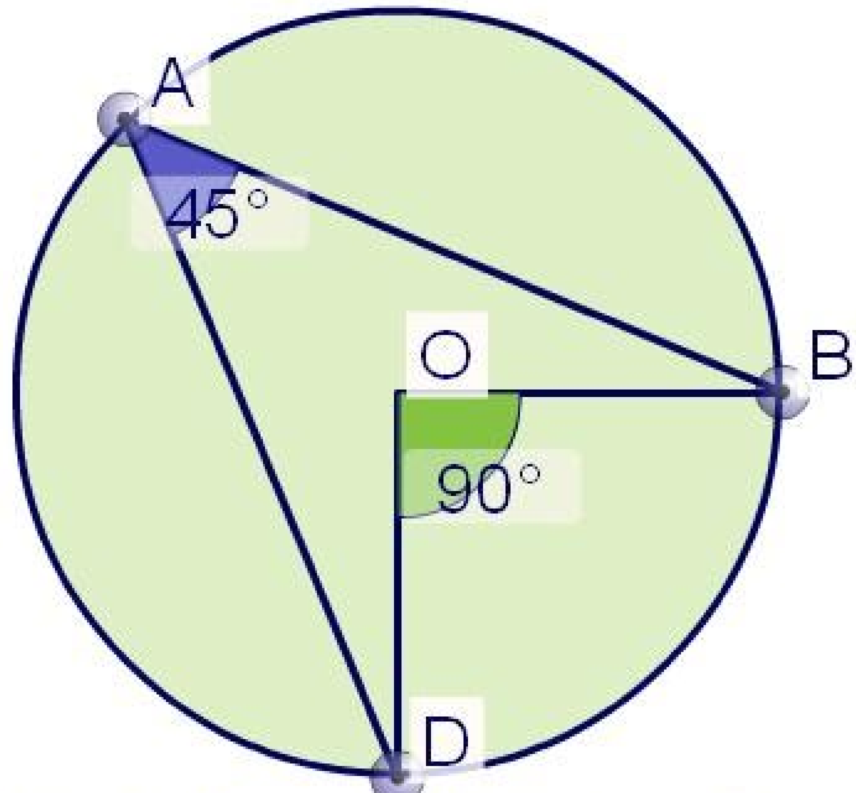
An **inscribed angle** is an angle formed by two chords of a circle whose vertex is on the circle.

For example, here the chords $\overline{AB}$ and $\overline{BC}$ meet at the point B. $\angle ABC$ is an inscribed angle.



The two chords also mark out an arc on the circle, $\widehat{AC}$. This is called the **intercepted arc**. This arc, and the unmarked chord $\overline{AC}$, are said to **subtend** $\angle ABC$.

The measure of the intercepted arc is the measure of the **central angle**, the angle formed at the center by the radii to A and C.

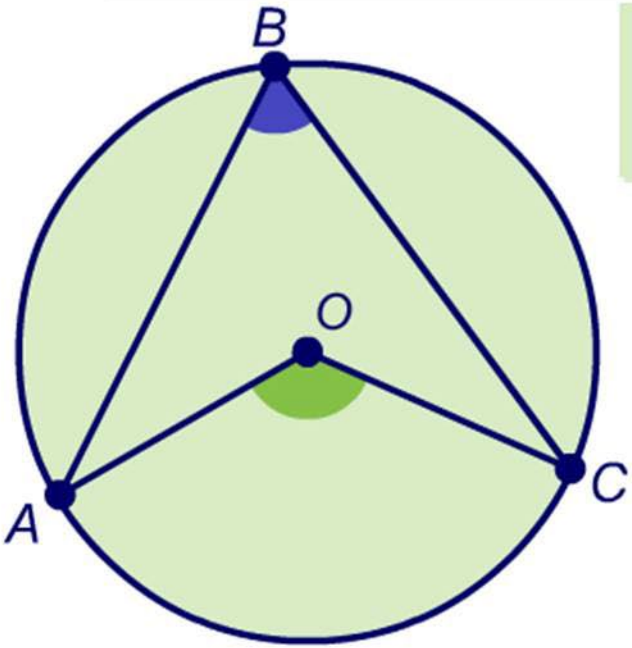


Drag the points around the circumference.

How does intercepted arc relate to the inscribed angle?

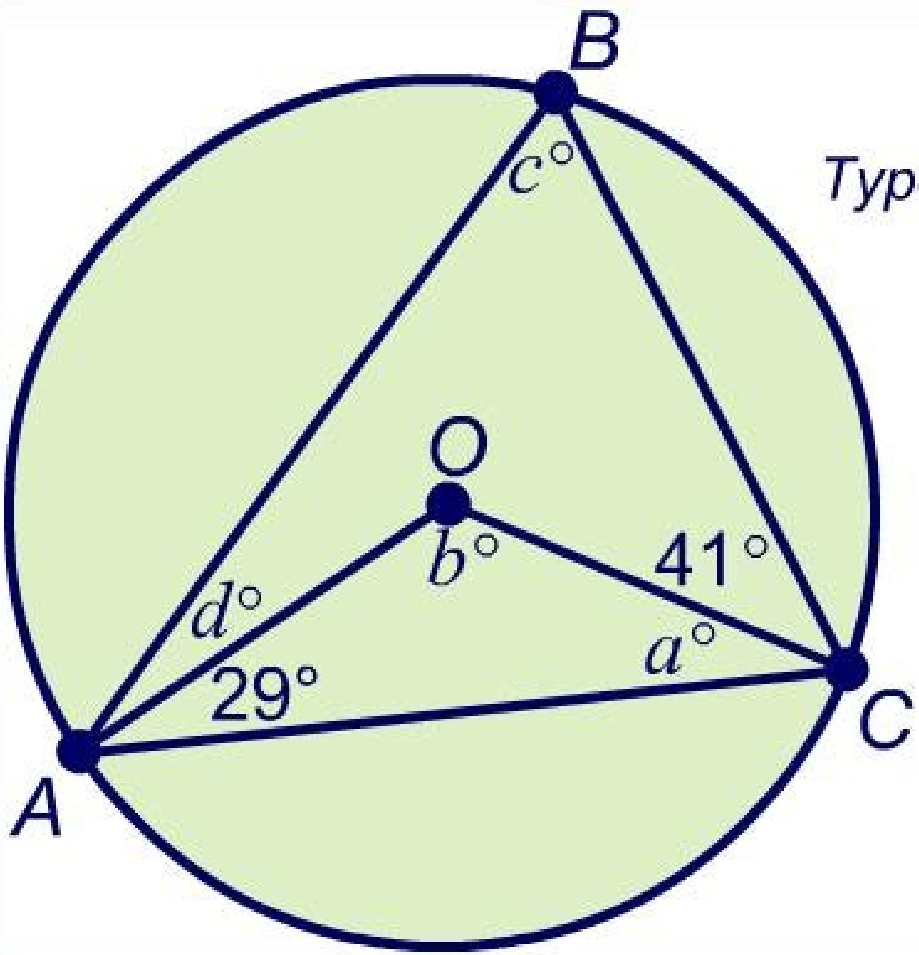
Proof of the Inscribed Angle Theorem

The Inscribed Angle Theorem: the measure of an inscribed angle is twice the measure of its intercepted arc.



Prove the theorem above for the case when O is inside the triangle ABC.

Press next to see the proof step-by-step.



Calculate the angles

Type the angles into the text boxes.

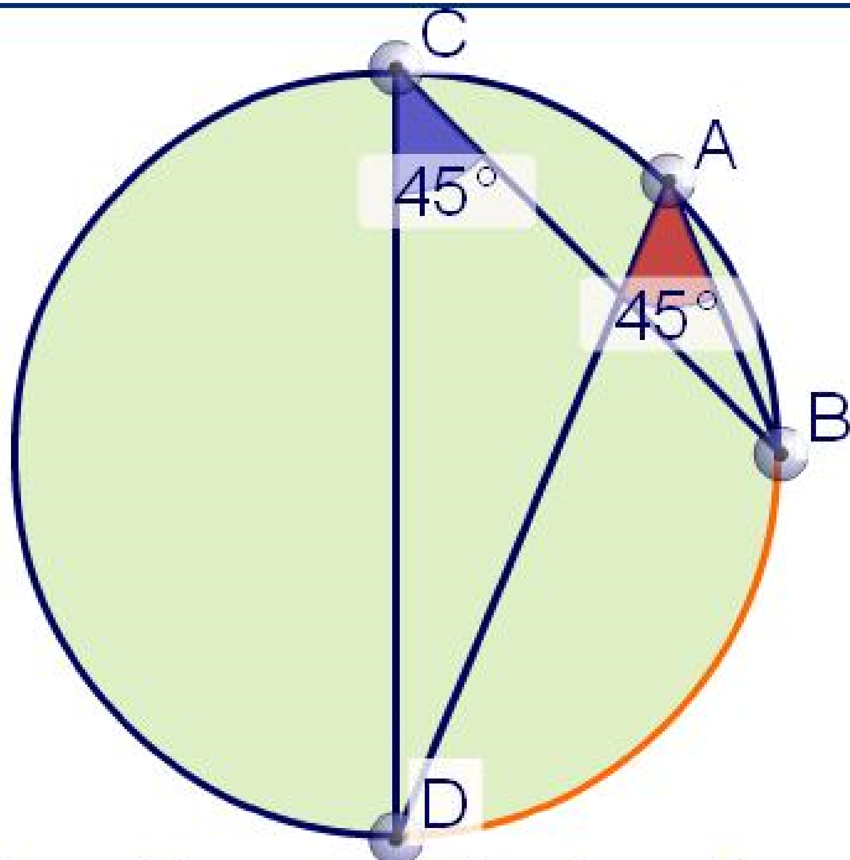
$a°$ = ________ °

$b°$ = ________ °

$c°$ = ________ °

$d°$ = ________ °

Comparing inscribed angles

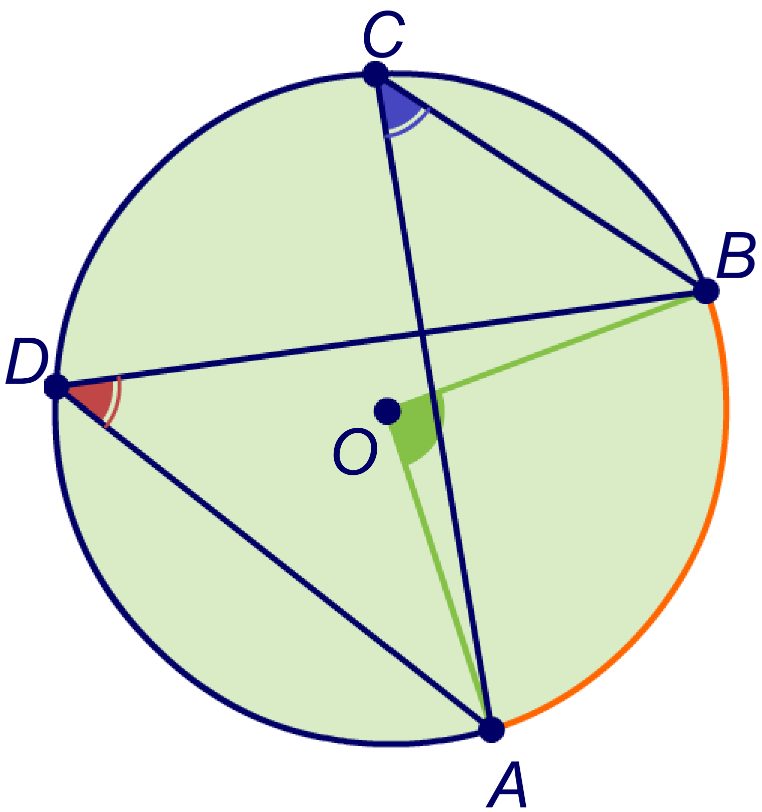


Drag the points around the circumference.

What do you notice about the two angles?

Congruent inscribed angles

Prove that if inscribed angles intercept the same arc, they are congruent.



By the ***inscribed angle theorem***, the measure of an inscribed angle is half the measure of its intercepted arc.

this gives:

$$m\angle ADB = \tfrac{1}{2}m\overset{\frown}{AB}$$

and

$$m\angle ACB = \tfrac{1}{2}m\overset{\frown}{AB}$$

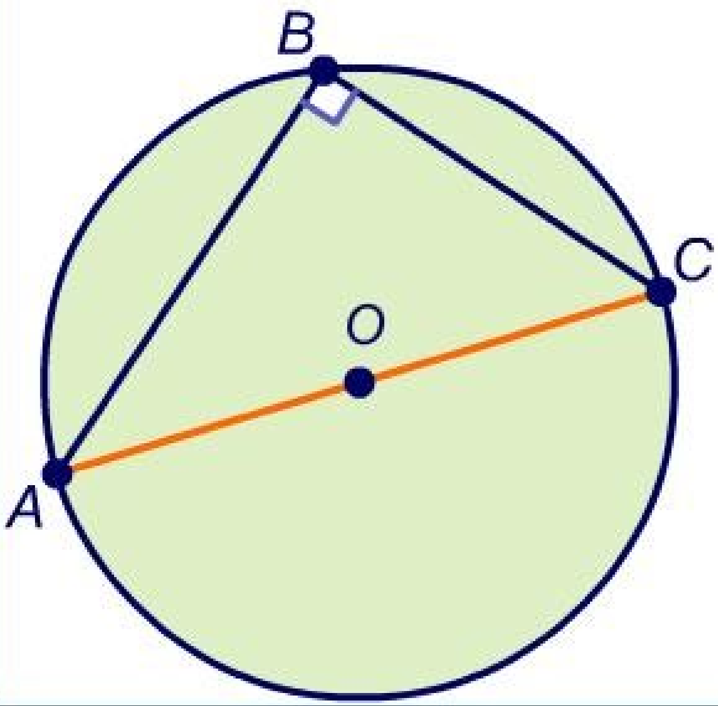
equate the two expressions:

$$m\angle ADB = m\angle ACB$$

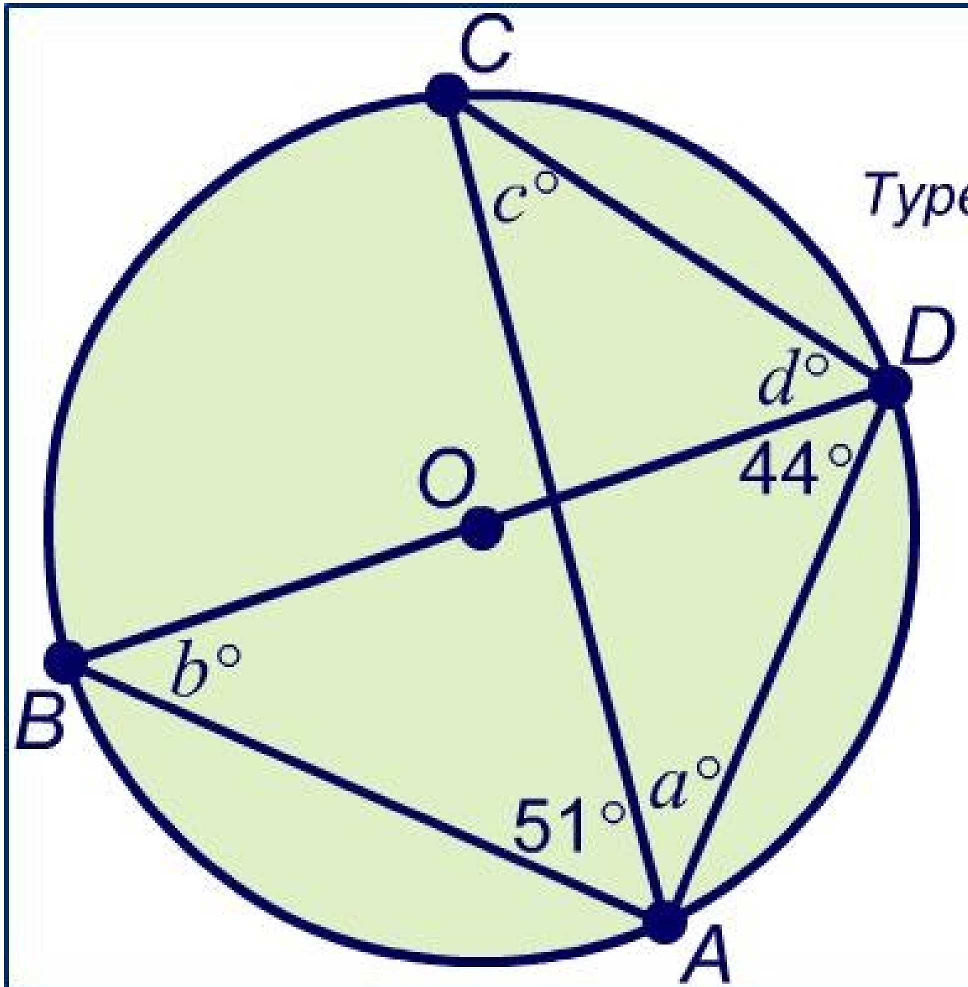
$$\angle ADB \cong \angle ACB$$

Prove the following theorem

An inscribed angle on a circle is a right angle ***if and only if*** it subtends a diameter of the circle.



Press the "=" button to show the calculations step-by-step.



Calculate the angles

Type the angles into the text boxes.

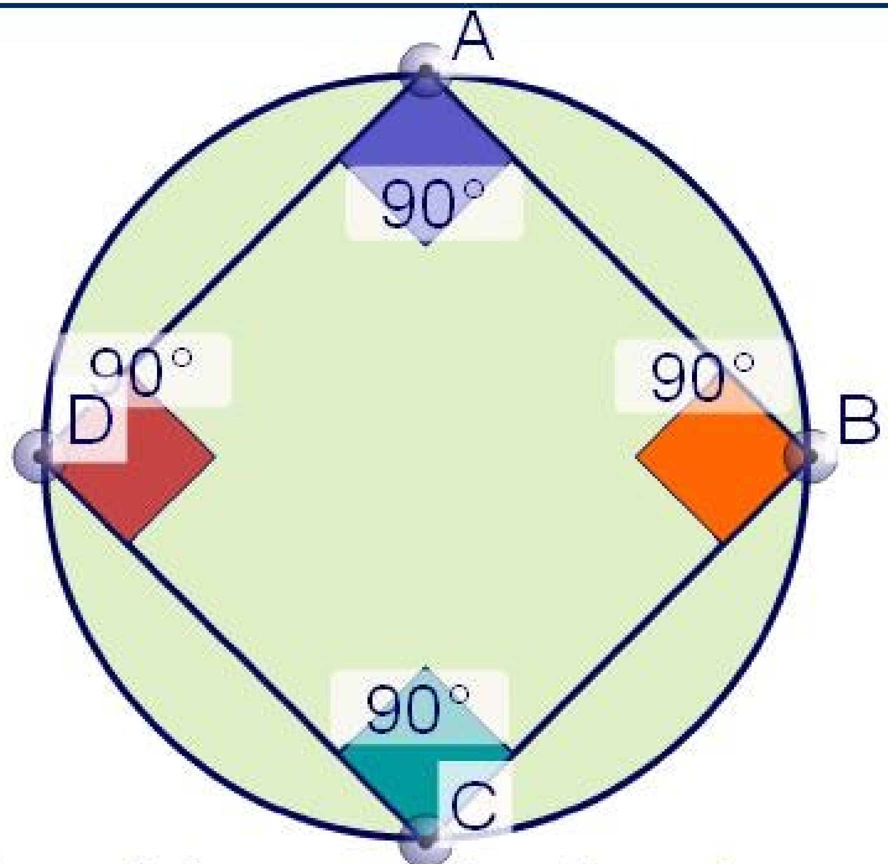
$a°$ = ________ °

$b°$ = ________ °

$c°$ = ________ °

$d°$ = ________ °

An inscribed quadrilateral



Drag the points around the circumference.

Add the opposite angles together. What do you notice?

Angles in an inscribed quadrilateral

The opposite angles in a quadrilateral inscribed in a circle add up to 180°.

Complete the paragraph proof of the theorem above.

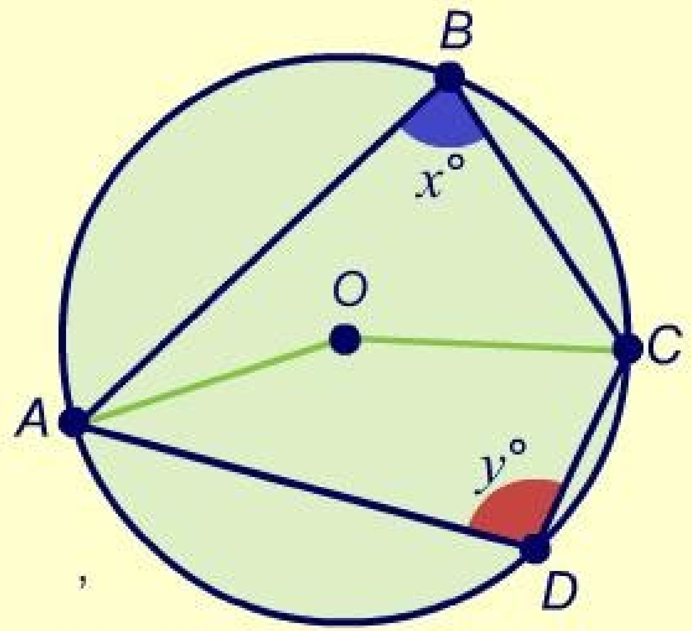
Write $x°$ for $m\angle ABC$ and $y°$ for $m\angle ADC$.

Mark the center of the circle, O, and draw the $\overline{AO}$ and $\overline{OC}$.

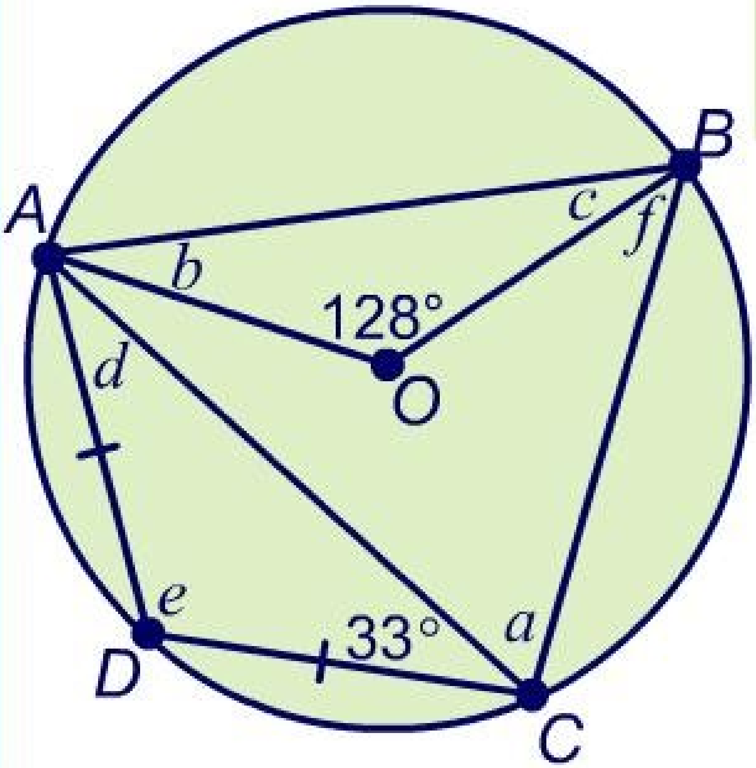
The angles give the measure of the intersected arc of $\angle ABC$ and the intersected arc of $\angle ADC$.

By ,

the angles at the center are $2y°$ and $2x°$



Calculate the angles



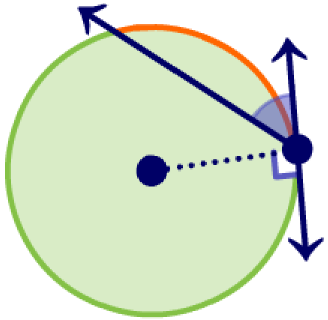
Question: 1/6

Find the measure of $\angle a$.

Press the "=" button to show the work step-by-step.

64°	57°
68°	60°

If an angle is formed on a circle between a chord and a tangent, by the *inscribed angle theorem*, the measure of the angle is half of the measure of its intercepted arc.



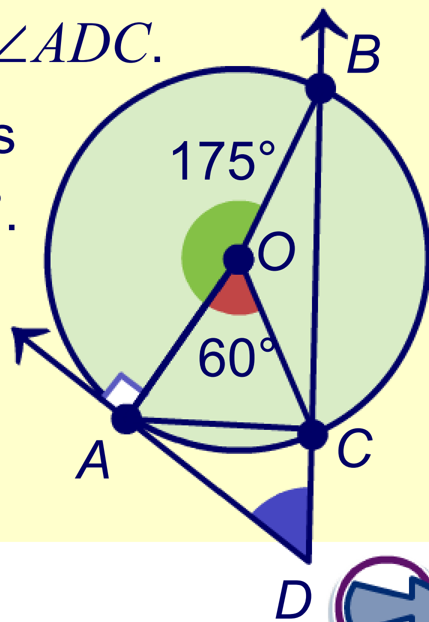
What happens if the angle is outside the circle?
What is the measure of the angle formed at D below?

By the exterior angle theorem, $m\angle ACB = m\angle CAD + m\angle ADC$.

$\angle CAD$ is an inscribed angle on a tangent, so $m\angle CAD$ is half the measure of its inscribed arc, 60°. $m\angle CAD$ = 30°.

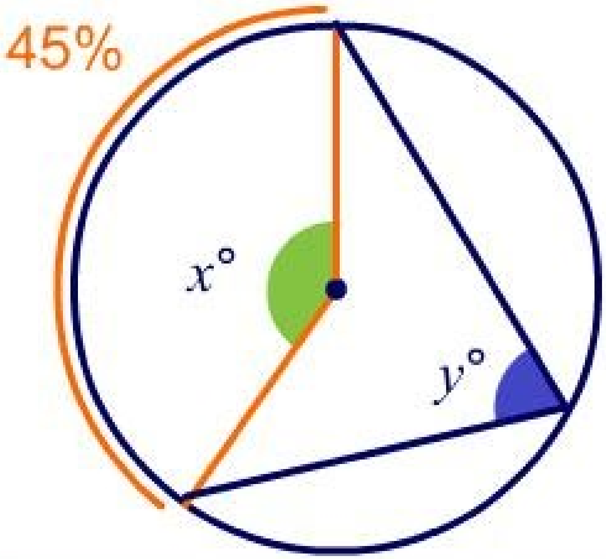
$\angle ACB$ is an inscribed angle, so $m\angle ACB$ is half of the measure of its inscribed arc, 175°. $m\angle ACB$ = 87.5°

$m\angle ADC = m\angle ACB - m\angle CAD$ = 87.5° – 30° = 57.5°



City problems

A city in a valley has a roughly circular perimeter. Two rivers enter the city at separate points and join together where they exit the city. 45% of the circumference of the city is between the two rivers. **What angle is formed where the rivers meet?**



Press the "=" button to show the calculations step-by-step.