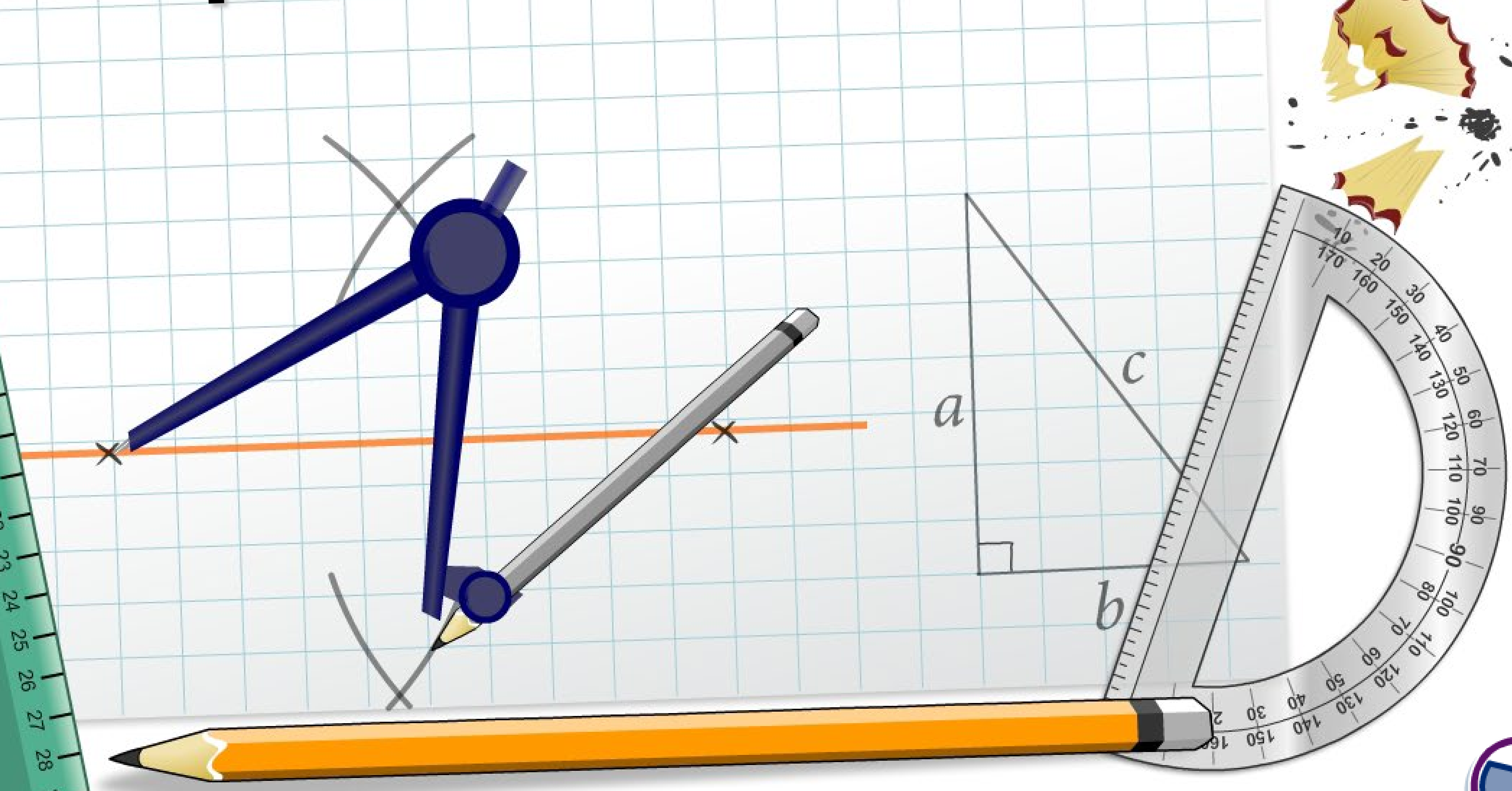
Equation of a Circle



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

MODELING

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

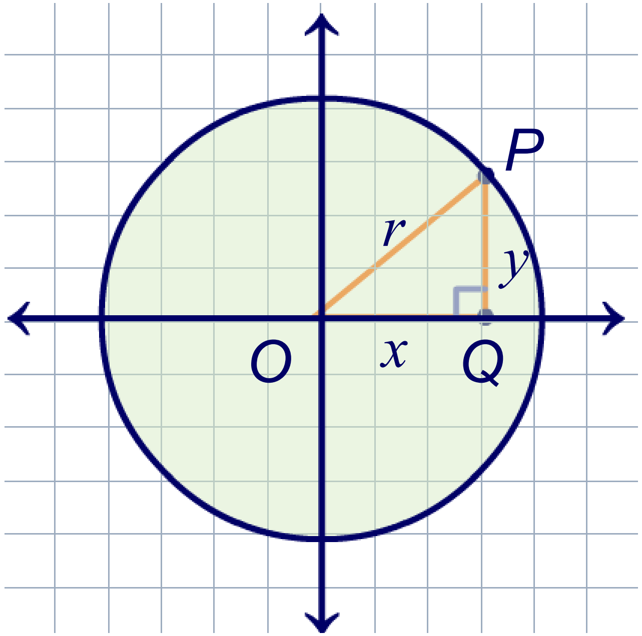
This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

The equation of a circle

A circle is drawn with its center at the origin O and radius r.

Show that the equation of the circle is $x^2 + y^2 = r^2$.



For any point $P(x, y)$ on the circle, $OP = r$.

If Q is the point $(x, 0)$ forming a right triangle with O and P, then $OQ = x$ and $PQ = y$.

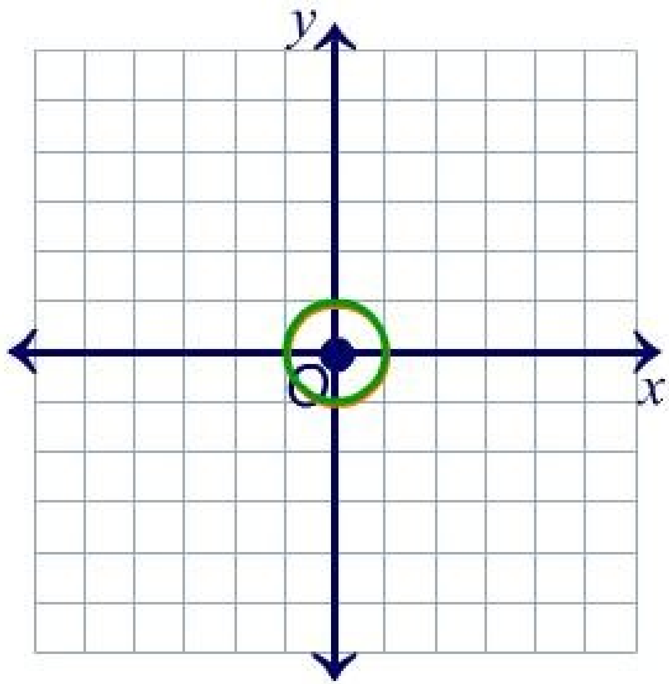
use the Pythagorean Theorem:

$$OQ^2 + PQ^2 = OP^2$$

so:

$$x^2 + y^2 = r^2$$

Transfomations of a circle



Change the radius and drag the orange circle to explore transformations of a circle.

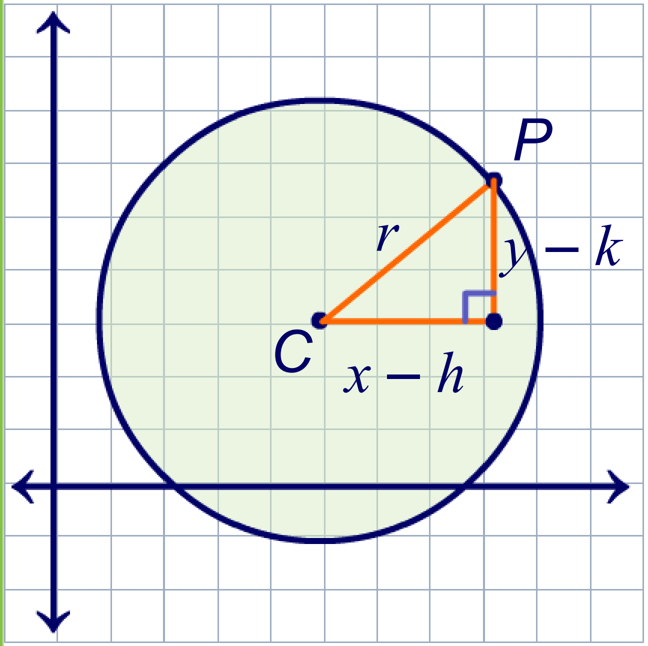
$$x^2 + y^2 = 0^2$$

$$(x + 0)^2 + (y + 0)^2 = 1^2$$

The equation of a circle

Suppose a circle is not centered at the origin, but the center C is the point (h, k).

Prove the equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$



For any point $P(x, y)$ on the circle, $CP = r$.

The vertical distance from C to P is $y - k$.

The horizontal distance from C to P is $x - h$.

use the Pythagorean Theorem:

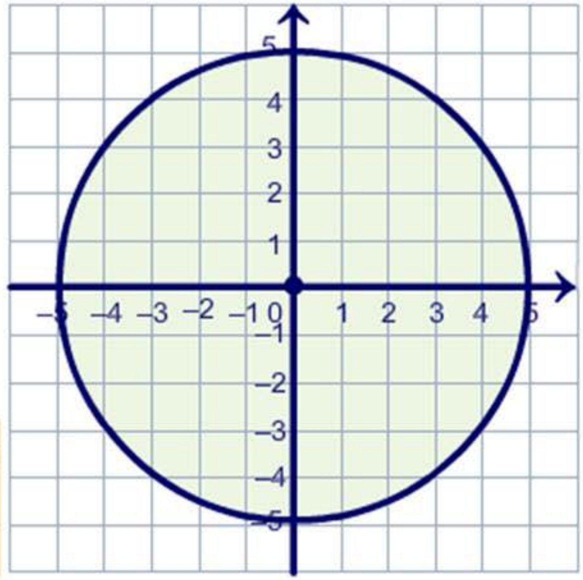
$$(x - h)^2 + (y - k)^2 = CP^2$$

so: $$(x - h)^2 + (y - k)^2 = r^2$$

Find the equation

What is the equation of the circle?

Question 1/5:



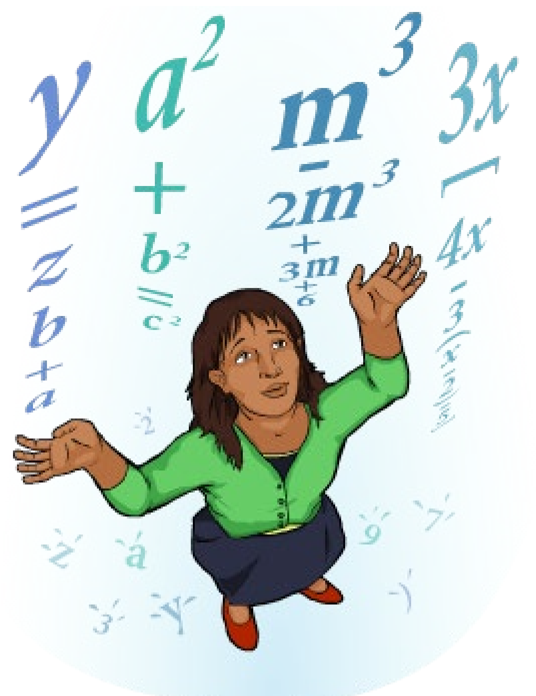
A) $(x + 1)^2 + (y - 1)^2 = 5$

B) $x^2 + (y + 1)^2 = 25$

C) $x^2 + y^2 = 5$

D) $x^2 + y^2 = 25$

The equation of a circle of radius r centered on the point (a, b) is $(x - a)^2 + (y - b)^2 = r^2$. This can be written in other forms.



expand the equation:

$$(x - a)^2 + (y - b)^2 = r^2$$

$$x^2 - 2ax + a^2 \quad + y^2 - 2by + b^2 \quad = r^2$$

rearrange:

$$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$$

since a, b and r are constants, write c for $a^2 + b^2 - r^2$:

$$x^2 + y^2 - 2ax - 2by + c = 0$$

Example

a) Write down the equation of the circle with center (3, 1) and radius $\sqrt{5}$.

b) Show that the point (4, –1) lies on the circle.

a) put the known information in the general equation of a circle:

$$(x - a)^2 + (y - b)^2 = r^2$$

$a = 3$, $b = 1$, and $r = \sqrt{5}$, so the equation of the circle is:

$$(x - 3)^2 + (y - 1)^2 = 5$$

b) The point (4, –1) lies on the circle if $x = 4$, $y = -1$ satisfies the equation of the circle.

substitute: $(4 - 3)^2 + (-1 - 1)^2 = 5$

$$1^2 + (-2)^2 = 5$$

$$1 + 4 = 5 \checkmark$$

Therefore, the point (4, –1) does lie on the circle.

The equation of a circle can be used to find the coordinates of its center and the length of its radius.

Find the center and the radius of a circle with the equation

$$(x - 2)^2 + (y + 7)^2 = 64$$

Compare this to the general form of the equation of a circle of radius r centered on the point (a, b):

$$(x - a)^2 + (y - b)^2 = r^2$$

$$(x - 2)^2 + (y + 7)^2 = 64$$

So $a = 2$, $b = -7$ and $r^2 = 64$.

The center is at the point (2, –7) and the radius is 8.

Find the center and the radius of a circle with the equation

$$x^2 + y^2 + 4x - 6y + 9 = 0$$

Complete the square to write the equation in the form $(x - a)^2 + (y - b)^2 = r^2$.

rearrange the equation to put the x terms and the y terms together:

$$x^2 + 4x + y^2 - 6y + 9 = 0$$

complete the square for the x terms and then for the y terms:

$$x^2 + 4x = (x + 2)^2 - 4 \qquad y^2 - 6y = (y - 3)^2 - 9$$

substitute these expressions into the equation and simplify:

$$(x + 2)^2 - 4 + (y - 3)^2 - 9 + 9 = 0$$

$$(x + 2)^2 + (y - 3)^2 = 4$$

$$(x + 2)^2 + (y - 3)^2 = 2^2$$

The center is at the point (–2, 3) and the radius is 2.

Match the equation to the center and radius of the circle

$x^2 + y^2 - 6y - 16 = 0$	$(0, 3)$ and $r = 5$
$x^2 + y^2 - 12x + 14y + 84 = 0$	$(-1, 9)$ and $r = 10$
$x^2 + y^2 + 8x - 4y + 11 = 0$	$(6, -7)$ and $r = 1$
$x^2 + y^2 - 4x + 6y - 276 = 0$	$(-4, 2)$ and $r = 3$
$x^2 + y^2 + 2x - 18y - 18 = 0$	$(2, -3)$ and $r = 17$