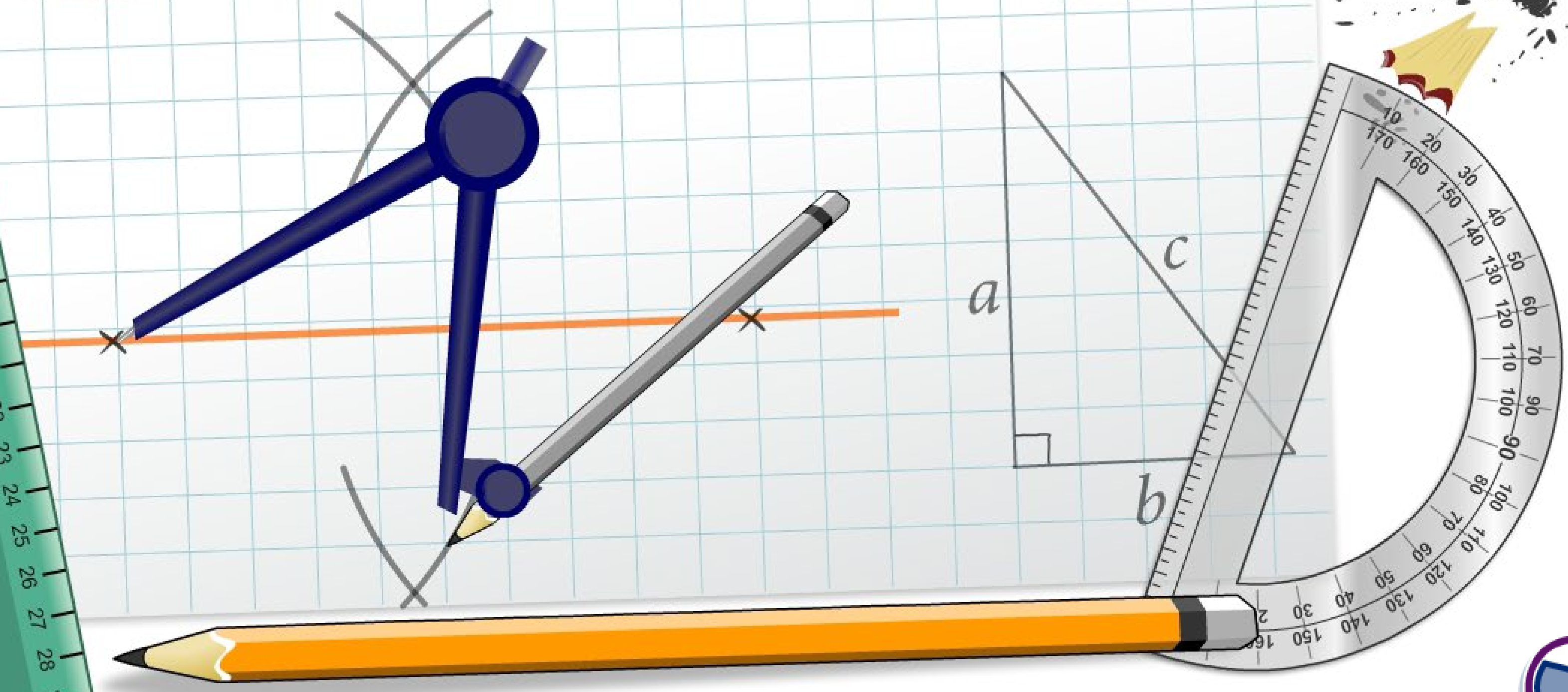
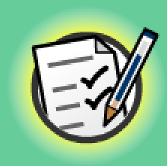
Independent Events

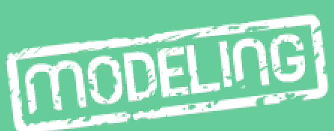


Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Independent events

Peter rolls an unbiased six-sided die fifty times and doesn't roll a six once. He says, "I must get a six soon!"

Is Peter correct?

No. Each roll of the die is unaffected by the previous outcomes, so the next roll of the die is no more likely to be a six than any of the other rolls. The probability is still $\frac{1}{6}$.

This is an example of **independent** events.

Events are independent if the outcome of one has no effect on the outcome of the other.

Independent or dependent?

Decide whether the events in each scenario are dependent or independent.

Press **start** to begin.

start

Combining independent probabilities

To find the probability of two independent events, their separate probabilities are multiplied together:

$$P(A \cap B) = P(A) \times P(B)$$

This rule applies only when events are independent. It also applies to multiple independent events. For example:

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

When rolling a standard die, what is the probability of rolling a 6 five times in a row?

$$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \mathbf{\frac{1}{36}}$$

When rolling a dice twice, what is the probability the first roll will be a one and second will be an even number?

method 1: use the table

number of possible results: 36

number of successful results: 3

$$P(1 \cap 2) = \frac{3}{36} = \mathbf{\frac{1}{12}}$$

method 2: use the formula

multiply the probability of each roll,
P(rolling a 1) × *P*(rolling an even number):

$$\frac{1}{6} \times \frac{3}{6} = \frac{3}{36} = \mathbf{\frac{1}{12}}$$

second roll (columns); first roll (rows)

	1	**2**	**3**	**4**	**5**	**6**
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Independent events

Use the table and formula to find the probability of getting a 1 on the first roll and an even number on the second.

second roll (columns); first roll (rows)

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Use the formula.

$$P(1 \cap \text{even}) = P(1) \times P(\text{even})$$

$$= \frac{1}{6} \times \frac{3}{6}$$

$$= \frac{3}{36} = \frac{\mathbf{1}}{\mathbf{12}}$$

Use the table.

$$\frac{\text{successful outcomes}}{\text{possible outcomes}} = \frac{3}{36} = \frac{\mathbf{1}}{\mathbf{12}}$$

Rachel draws a playing card randomly from a full deck.

Matt then throws a fair ten-sided die.

Press on one of Rachel’s events and one of Matt’s events and find the probability that both events happen.

Press **start** to begin.

start

This is the result of a survey of student grade and hair color:

grade	blonde	brown	red	other	total
9th	38	25	5	17	85
10th	41	28	4	18	91
11th	42	28	5	18	93
total	121	80	14	53	269

Use the rule $P(A \cap B) = P(A) \times P(B)$ to decide if P(brown ∩ 11th grade) are independent events or not.

prove P(brown ∩ 11th *grade*) = P(brown) × P(11th grade)

$$P(\text{brown} \cap 11^{th}\text{ grade}) = \frac{28}{269} = 0.10$$

$$P(\text{brown}) \times P(11^{th}\text{ grade}) = \frac{80}{269} \times \frac{93}{269} = 0.10$$

These events are independent.

Frequency tables

grade	blonde	brown	red	other	total
9th	38	25	5	17	85
10th	41	28	4	18	91
11th	42	28	5	18	93
total	121	80	14	53	269

What is P(blonde ∩ 10^{th} grade)?

find the P(10^{th} grade): 91 ÷ 269 = 0.34

find the P (blonde): 121 ÷ 269 = 0.45

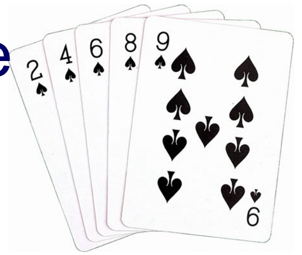
find P (blonde ∩ 10^{th} grade): 0.34 × 0.45 = **0.15**

or divide the instances of blonde 10^{th} graders by total students: 41 ÷ 269 = **0.15**

What is P(11^{th} grade ∩ not red haired)?

Tree diagrams for independent events

Sarah has fifteen playing cards left from an incomplete pack. Six of the cards are red and the rest are black.



What is the decimal probability of choosing a red card at random from the pack?

$$P(\text{red card}) = \frac{6}{15} = \mathbf{0.4}$$

Sarah picks a card at random from the pack, replaces it and then picks another. If she picks two red cards, she will go to a party. Otherwise, she will stay in and do homework.

What is the probability that Sarah will go to the party and the probability that she will stay in to do homework?

If Sarah picks two red cards she will go to a party. Otherwise she will stay in and do homework.

Use the calculator to work out the probability of each set of events. Press the reveal buttons to show the answers.

Press **start** to begin.

start

Tree diagrams for independent events

Georgia passes through three sets of traffic lights on her drive to work.

By repeating the journey many times, she has found the probability of having to stop at each set of lights:

- 1^{st} set: 0.6
- 2^{nd} set: 0.1
- 3^{rd} set: 0.2

Illustrate the situation using a tree diagram. What is the probability that Georgia has to stop at least twice on her way to work?

Press the **Stop** and **Go** buttons to reveal the probability of that event occuring. Use the calculator to work out the probability of each set of events then press the **reveal** buttons to display the answer.

What is the probability that Georgia will stop at two red lights on her way to work?

Press **play** to start.

start

Proving independent events

Use the formula to prove that stopping at the first red light and the third red light are independent events, $P(1^{st} \text{ red} \cap 3^{rd} \text{ red}) = P(1^{st} \text{ red}) \times P(3^{rd} \text{ red})$.

probability of stopping at the 1st red:	$P(1^{st} \text{ red}) = 0.6$
probability of stopping at the 3rd red:	$P(3^{rd} \text{ red}) = 0.2$
find all outcomes where she stops at the 1st and 3rd red lights:	$P(\text{RRR})$ and $P(\text{RGR})$
add the probabilities of these outcomes:	$0.012 + 0.108 = \mathbf{0.12}$
evaluate:	$0.6 \times 0.2 = \mathbf{0.12}$

$$P(1^{st} \text{ red} \cap 3^{rd} \text{ red}) = P(1^{st} \text{ red}) \times P(3^{rd} \text{ red}) \quad ✓$$