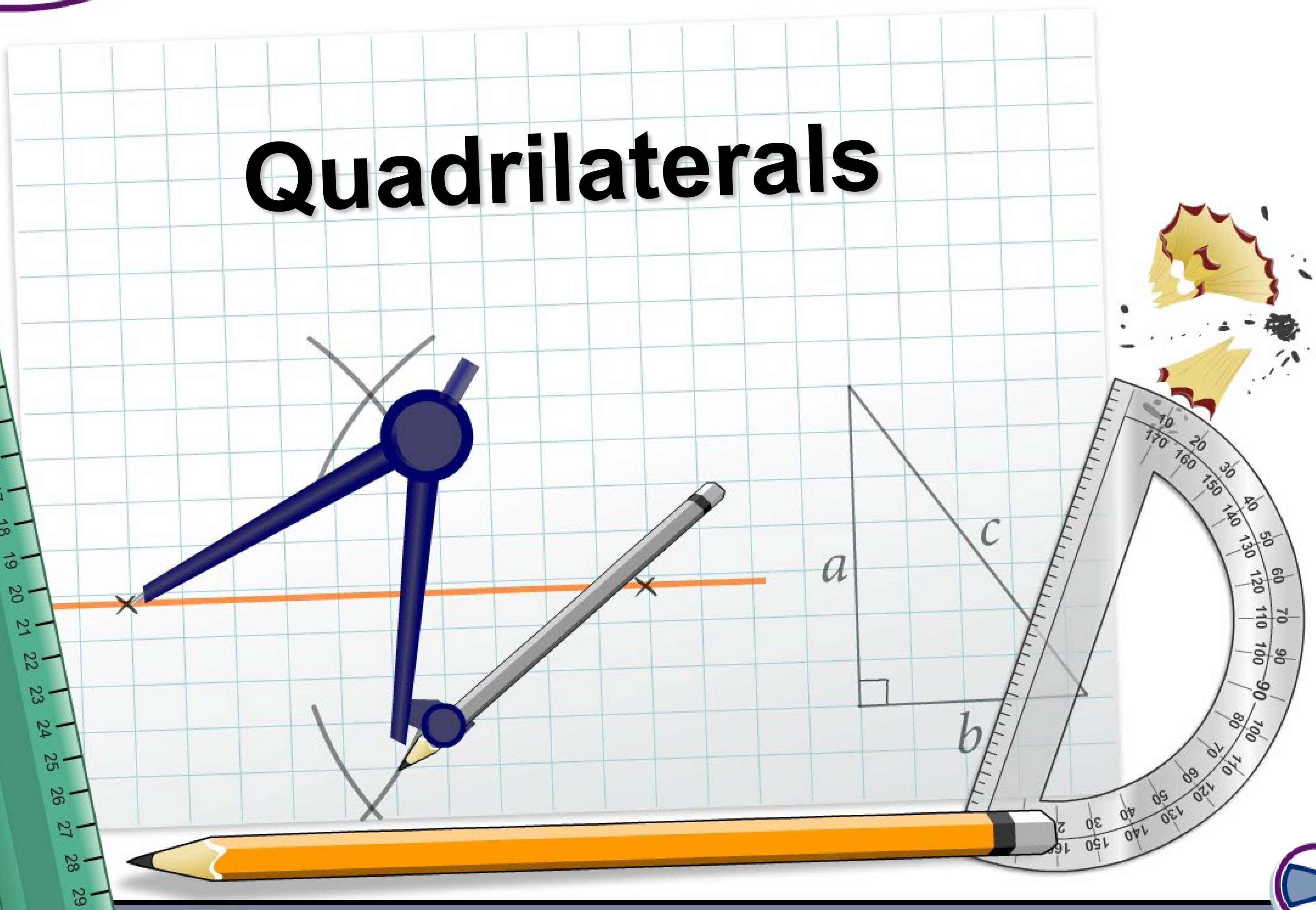
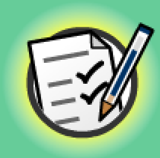
Quadrilaterals



Information

Common core icons



This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**

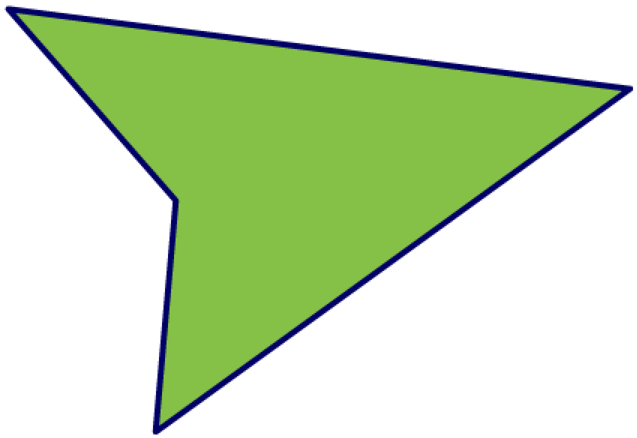
This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

What is a quadrilateral?

A **quadrilateral** is a four-sided polygon.

A quadrilateral is the polygon with the fewest number of sides that allows for it to be a **concave** polygon (where one of the interior angles is greater than 180 degrees).



Why is it impossible to have a concave triangle?

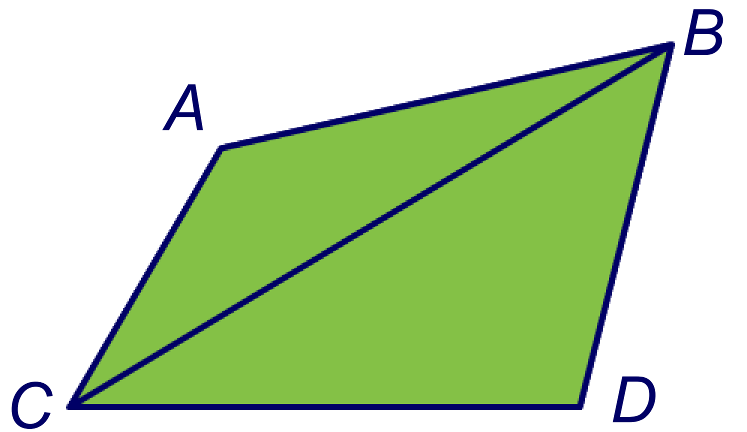
Since the interior angles of a triangle have to sum to 180°, there can not be an interior angle greater than 180 degrees.

Quadrilateral angle sum

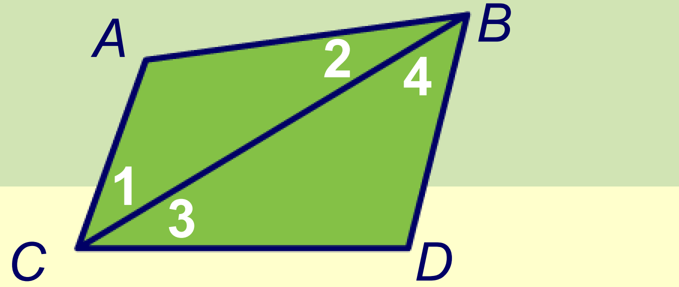
Polygon angle sum theorem:
The sum of the measures of the interior angles of a convex n-sided polygon is $(n - 2)\ 180°$

Using this theorem, we can see that the sum of the measures of a quadrilateral is: $2 \times 180° = 360°$.

The angle sum theorem of a quadrilateral can also be explained in a diagram. A quadrilateral can be divided into two triangles by drawing a line between two opposite vertices. Each of these triangles has an angle sum of 180 degrees.



Prove that the interior angle sum of quadrilateral *ABCD* is 360°.



given: quadrilateral $ABCD$, split by $\overline{BC}$

hypothesis: $m\angle A + m\angle B + m\angle C + m\angle D = 360°$

triangle angle sum theorem: $m\angle A + m\angle 1 + m\angle 2 = 180°$ and $m\angle D + m\angle 3 + m\angle 4 = 180°$

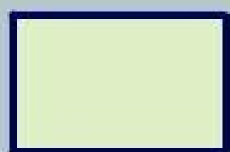
angle addition: $m\angle 1 + m\angle 3 = m\angle C$ and $m\angle 2 + m\angle 4 = m\angle B$

angle substitution: $m\angle A + m\angle B + m\angle C + m\angle D$
$= m\angle A + (m\angle 2 + m\angle 4) + (m\angle 1 + m\angle 3) + m\angle D$

group by triangles: $(m\angle A + m\angle 1 + m\angle 2) + (m\angle D + m\angle 3 + m\angle 4)$

triangle angle sum theorem: $m\angle A + m\angle B + m\angle C + m\angle D = 180° + 180° = 360°$ ✓

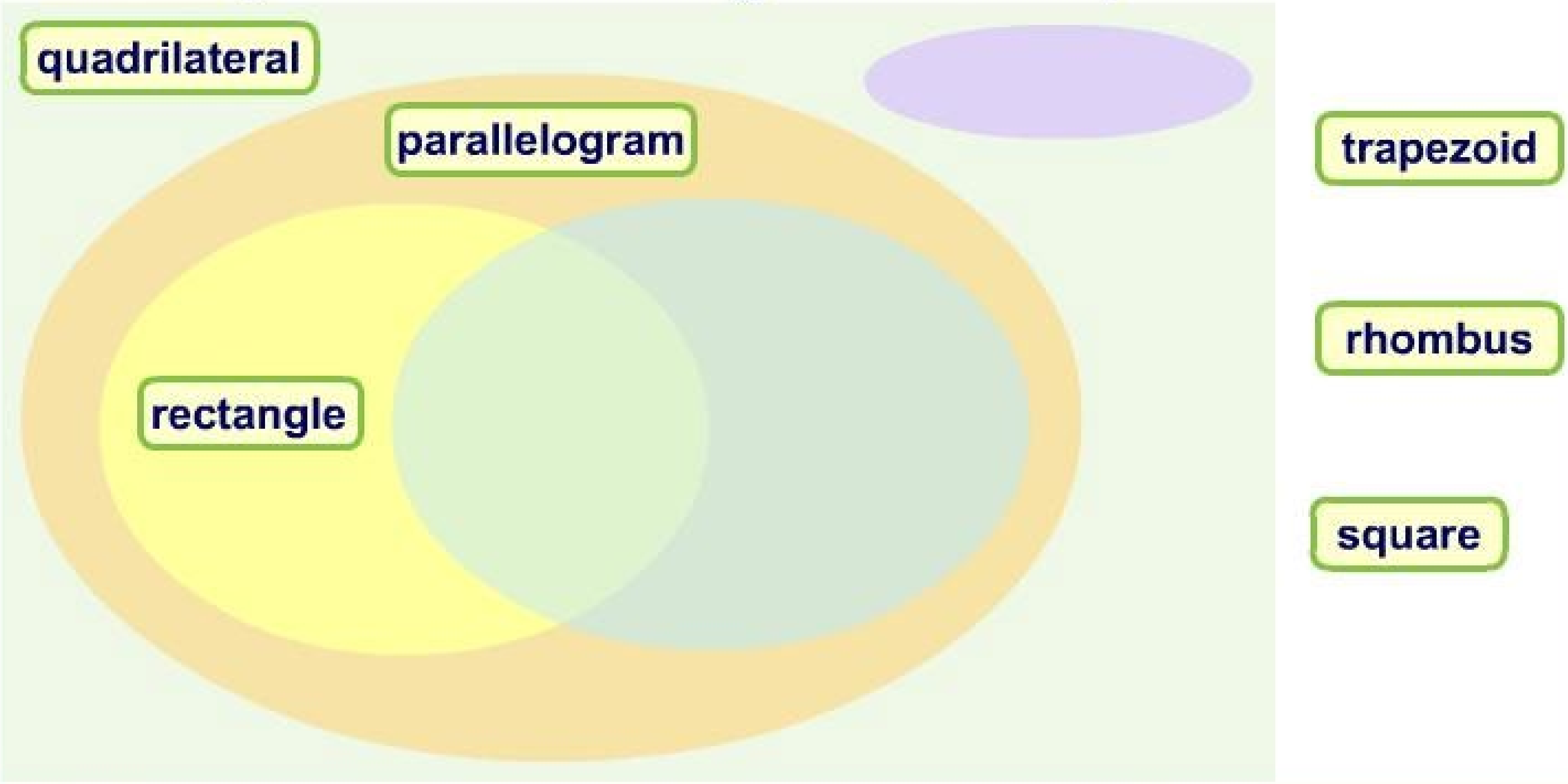
Complete the table for the quadrilaterals

	rectangle	rhombus	parallelogram	trapezoid
description				
diagram				

all sides are congruent

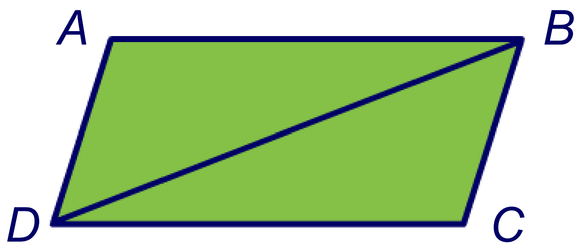
Types of parallelograms

Complete the Venn diagram for the quadrilaterals



In parallelogram $ABCD$ we know that $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{CD}$.

Diagonal $\overline{BD}$ divides the parallelogram into two triangles.

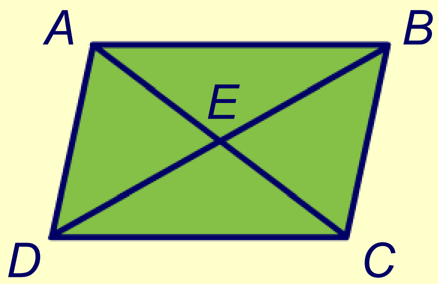


Use parallelogram $ABCD$ to prove that opposite sides and opposite angles in any parallelogram are congruent.

alternate interior angles are congruent:	$\angle ADB \cong \angle DBC$ and $\angle ABD \cong \angle BDC$
reflexive property:	$\overline{BD} \cong \overline{BD}$
ASA property:	$\triangle ABD \cong \triangle BCD$
corresponding parts of congruent triangles are congruent (CPCTC):	$\overline{AD} \cong \overline{CB}$ and $\overline{AB} \cong \overline{CD}$ ✓ Also $\angle BAD \cong \angle BCD$ ✓

A **bisector** is a line that goes through the midpoint of a segment and divides the line into two congruent parts.

Given parallelogram $ABCD$, with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E, prove that $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$ (that the diagonals of the parallelogram bisect each other)



congruence of alternate interior angles:	$\angle ABD \cong \angle BDC$ and $\angle CAB \cong \angle ACD$
congruence of opposite sides:	$\overline{AB} \cong \overline{CD}$
ASA property:	$\triangle ABE \cong \triangle CDE$
corresponding parts of congruent triangles are congruent (CPCTC):	$\overline{BE} \cong \overline{DE}$ and $\overline{AE} \cong \overline{CE}$ ✓

Parallelogram properties

Sort the statements about parallelograms as true or false.

true	false

All parallelograms have no lines of symmetry

Proving that ABCD is a parallelogram

If *ABCD* is a quadrilateral, then how can we prove that it is also a parallelogram?

We must prove that both pairs of opposite sides are parallel.

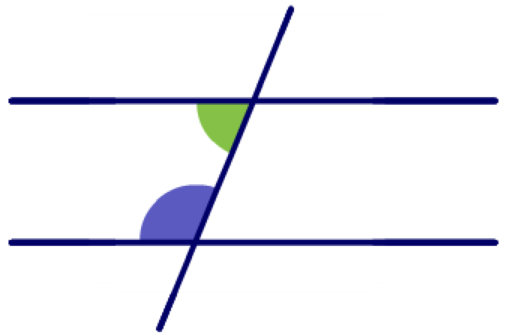
How can we prove that lines are parallel in a quadrilateral?

To prove that lines are parallel,
we must prove one of the following:

1) Alternate interior angles are congruent.

2) Corresponding angles are congruent.

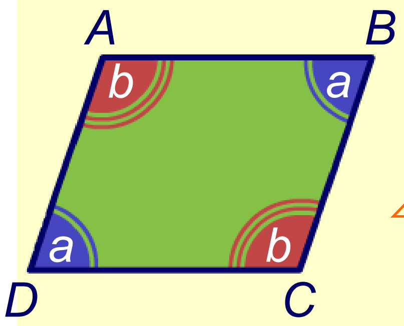
3) Same side interior angles are supplementary.



Proving that ABCD is a parallelogram

If the opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

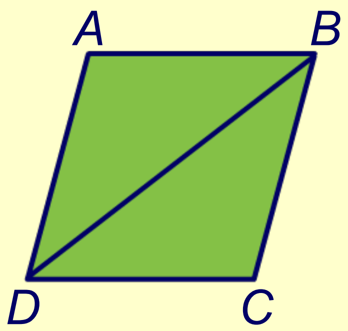
Given quadrilateral $ABCD$ with $\angle A \cong \angle C$ and $\angle B \cong \angle D$, prove that $ABCD$ is a parallelogram.



polygon angle sum theorem:	$m\angle A + m\angle B + m\angle C + m\angle D = 360°$
$\angle A \cong \angle C$ and $\angle B \cong \angle D$:	$a + a + b + b = 360°$
group like terms:	$2a + 2b = 360°$
divide by 2:	$a + b = 180°$
a and b are supplementary:	$\angle A$ and $\angle D$ are supplementary $\angle D$ and $\angle C$ are supplementary
converse of the alternate interior angle theorem:	$\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$ ✓

Proof of a parallelogram

With a partner, use congruent triangles to prove that if the opposite sides of a quadrilateral are congruent, then it is a parallelogram.

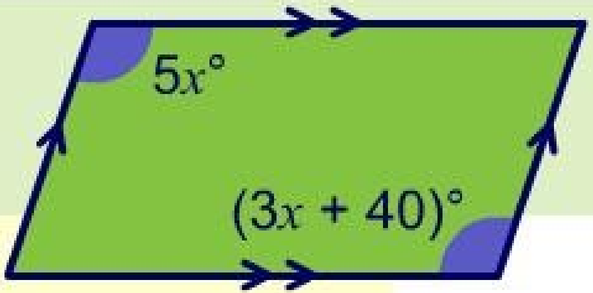


given:	quadrilateral $ABCD$ with $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$
hypothesis:	$ABCD$ is a parallelogram
reflexive property:	$\overline{BD} \cong \overline{BD}$
SSS congruence postulate:	$\triangle ABD \cong \triangle CBD$
CPCTC:	$\angle ABD \cong \angle BDC$ and $\angle CBD \cong \angle ADB$
converse of the alternate interior angle theorem:	Since $\angle ABD \cong \angle BDC$, $\overline{AB} \parallel \overline{CD}$ Since $\angle CBD \cong \angle ADB$, $\overline{AD} \parallel \overline{BC}$
given $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$:	$ABCD$ is a parallelogram. ✓

Parallelogram side and angle calculations

Question: 1/2

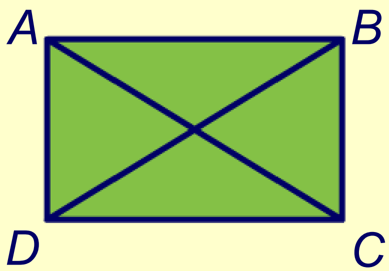
Solve for x in this parallelogram:



Press the "=" button to show the work step-by-step.

- 8°
- 10°
- 40°
- 20°

Prove that if the diagonals of a parallelogram are congruent, then the parallelogram must be a rectangle.



given:	parallelogram $ABCD$ where $\overline{AC} \cong \overline{BD}$
hypothesis:	$ABCD$ is a rectangle
congruence of opposite sides of a parallelogram:	$\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{CD}$
SSS congruence postulate:	$\triangle ADC \cong \triangle BCD$
CPCTC:	$\angle ADC \cong \angle BCD$
since $AD \parallel BC$ and they are same side interior angles:	$\angle ADC$ and $\angle BCD$ are supplementary
$\angle ADC$ and $\angle BCD$ are congruent and supplementary:	$m\angle ADC = m\angle BCD = 90°$ ✓